Three-Cornered Hat Method via GPS Common-View Comparisons

Diego Luna, Daniel Pérez, Alejandro Cifuentes, and Demián Gómez

***Abstract*— This paper evaluates the stability of three industrial cesium clocks located in different sites, comparing them via the common-view technique and using the three-cornered hat (TCH) method. Validation of the implementation is obtained by comparing results with Coordinated Universal Time (UTC) and Rapid UTC values reported for the three cesium clocks involved. An enhanced TCH method is presented and implemented, yielding corrections up to 30% in the Allan deviation estimates. The main feature of the developed method is the possibility of computing the absolute stability of remote clocks at averaging times of about 2 h.**

***Index Terms*— Atomic clocks, frequency stability, global positioning system (GPS), noise measurement, time-domain analysis.**

I. Introduction

Atomic clocks are a key component in different technological fields. Besides their use in time-keeping laboratories, several applications like navigation systems, telecommunications, astronomy, and radar use them as time and frequency references. Therefore, assessment of high-quality clocks performance is of great importance. In particular, phase and frequency instabilities are the most significant characteristics.

Global navigation satellite systems (GNSSs) are designed as a satellite-based radio navigation system. In geospatial applications, the GNSS are used to find the instantaneous position of the antenna of the receiver. In addition, GNSS may also be used for timing applications. For example, remote standards can be compared using the signals broadcasted by the GNSS. Common view (CV) is one of the available techniques for this purpose [1].

The CV method is based on the observation of the signal transmitted by a source and received at different locations. The concept of global positioning system (GPS) CV is as follows.

Consider two stations (i and j), which simultaneously measure the time difference between their local clocks and a received time signal t^k, from a GPS satellite k. The time differences between the station i and the satellite k and between the station j and the satellite k are thus expressed (after several delay corrections) according to

$$\delta t_i^k = t_i - t^k \quad (1)$$
$$\delta t_j^k = t_j - t^k. \quad (2)$$

With this in mind, the time difference $(t_i - t_j)$ between clocks can be calculated as

$$\delta t_{ij} = \delta t_i^k - \delta t_j^k = t_i - t_j. \quad (3)$$

As previously stated, (3) is a simplified expression of the GPS CV principle. In practice, measurements are corrected by several biases [2]: ionospheric and tropospheric delay, internal delays of the receiver, and so on.

Allan variance is the most common way to characterize the frequency stability of an oscillator in the time domain.

When measuring the phase difference between two oscillators $x(t)$, the results are related to the fractional frequency deviation by

$$\bar{y}_k = \frac{x(t_k + \tau) - x(t_k)}{\tau} \quad (4)$$

where the bar denotes the average over the measurement (sampling) interval τ.

The Allan variance [3] $\sigma_y^2(\tau)$ is defined as

$$\sigma_y^2(\tau) = \frac{1}{2}\langle(\bar{y}_{k+1} - \bar{y}_k)^2\rangle \quad (5)$$

where $\langle\ldots\rangle$ denotes the average over a large number of samples. In terms of phase differences, (5) can be expressed as

$$\sigma_y^2(\tau) = \frac{1}{2\tau}\langle(x(t_k + 2\tau) - 2x(t_k + \tau) + x(t_k))^2\rangle. \quad (6)$$

This original definition of the Allan variance (5) or (6) has been improved by its overlapping version, which makes a more efficient use of the data set. This modern version has a lower dispersion in the results and is the one implemented in this paper.

In practice, frequency stability measurements include noise contributions from both the device under test and the reference. In an optimal scenario, the reference noise is low enough to neglect its contribution in the characterization of the device under test. Another possibility is the so-called "three-cornered hat" (TCH) method for computing the individual variances of the clocks [3], [4].

Given a set of three pairs of measurements for three independent frequency sources 1–3, it is possible to calculate

Manuscript received February 16, 2017; accepted February 17, 2017. Date of publication April 3, 2017; date of current version July 12, 2017. The Associate Editor coordinating the review process was Dr. Dario Petri.

D. Luna and D. Pérez are with the INTI-Fisica y Metrologia Center, Buenos Aires, Argentina (e-mail: luna@inti.gob.ar).

A. Cifuentes is with the Observatorio Naval de Buenos Aires.

D. Gómez is with the Instituto Geográfico Nacional and The Ohio State University.

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TIM.2017.2684918

the individual variances as follows: the first step is to take a time series corresponding to the difference between each of the three clock pair combination and to form an estimate of the Allan deviation for each of the three time series. The variance of each pair will have contributions of both oscillators, so we can consider them as follows:

$$\sigma_{12}^2 = \sigma_1^2 + \sigma_2^2 \tag{7}$$
$$\sigma_{13}^2 = \sigma_1^2 + \sigma_3^2 \tag{8}$$
$$\sigma_{23}^2 = \sigma_2^2 + \sigma_3^2 \tag{9}$$

where σ_1, σ_2, and σ_3 correspond to the variance of each individual clock and no correlations between variances are considered. The original source variances [5] can be obtained by solving the system

$$\begin{aligned}\sigma_1^2 &= \frac{1}{2}\left(\sigma_{12}^2 + \sigma_{13}^2 - \sigma_{23}^2\right)\\ \sigma_2^2 &= \frac{1}{2}\left(\sigma_{12}^2 + \sigma_{23}^2 - \sigma_{13}^2\right)\\ \sigma_3^2 &= \frac{1}{2}\left(\sigma_{13}^2 + \sigma_{23}^2 - \sigma_{12}^2\right).\end{aligned} \tag{10}$$

The phase differences between clock pairs in TCH implementations are usually carried in single locations, i.e., clocks are compared *in situ*. Recently, a remote comparison to estimate TCH variances has been implemented over fiber networks [6]. In this paper, CV phase differences are used as data sources for TCH computation. To the best of our knowledge, there is no record of the combination of both techniques.

This paper evaluates the performance of three cesium clocks located in different facilities, combining the CV and the TCH techniques. An enhanced version of TCH method is implemented to account for the noise added by the remote comparison.

This paper is organized as follows. Section II proposes a new version of TCH, which considers a noise contribution independent of the clocks. Section III describes and validates the CV algorithm implemented in this analysis. The stability of the three clocks under test is discussed in Section IV. Transfer noise is computed in Section V by implementing the proposal of Section II to measured data. Finally, conclusions are presented in Section VI.

II. Transfer Noise Model

This section proposes an enhanced version of the TCH method. This extended version of TCH (eTCH) performs correction for the noise introduced in CV comparisons.

Equations (7)–(9) model the measured variances in the absence of correlations and measurement noise. If we consider noise sources, independent of the clocks, that are present in the three CV measurements, we can model this new contribution as

$$\begin{aligned}\sigma_{12}^2 &= \sigma_1^2 + \sigma_2^2 + \sigma_{\text{link\_12}}^2\\ \sigma_{13}^2 &= \sigma_1^2 + \sigma_3^2 + \sigma_{\text{link\_13}}^2\\ \sigma_{23}^2 &= \sigma_2^2 + \sigma_3^2 + \sigma_{\text{link\_23}}^2.\end{aligned} \tag{11}$$

The intercomparison of a set of three clocks yields the measured time differences δt_{12}, δt_{13}, and δt_{23}. Ideally, $t_{\text{closure}} \equiv \delta t_{13} - \delta t_{12} - \delta t_{23} = 0$ should hold for the measurements, since in principle, the variances of the individual clocks do not affect t_{closure} (the closure involves first differences, so oscillations in the clocks outputs are compensated in this estimator). Possible biases in the calibrations of the receivers also cancel out for the same reason. Generally, measurement noise avoids these time differences to perfectly compensate. In the particular case of CV comparisons, each of the time differences between clocks is in fact an average of the clocks differences over different sets of satellites, and the dispersion in the values of δt_{12}, δt_{13}, and δt_{23} avoids t_{closure} from being identically zero. These dispersions are generated mainly by residual errors from the ionospheric and tropospheric corrections, which are known to have an uncertainty [7] of the order of 2 ns, consistent with the typical standard deviations obtained in Fig. 3.

As the baselines involved in this work (~20 km) are much shorter than the orbits of the GPS satellites [8], we can consider that the contribution of atmospheric noise is common to the three receivers. To account for this noise source, we define $\sigma_{\text{link}}(\tau)$ as the Allan variance of the residual values t_{closure} and use it as a correction in the measurements to compensate for atmospheric noise.

It is straightforward to show that if we consider $\sigma_{\text{link\_12}}^2 \approx \sigma_{\text{link\_13}}^2 \approx \sigma_{\text{link\_23}}^2 \approx \sigma_{\text{link}}^2$, the individual variances are now expressed as

$$\begin{aligned}\sigma_1^2 &= \frac{1}{2}(\sigma_{12}^2 + \sigma_{13}^2 - \sigma_{23}^2 - \sigma_{\text{link}}^2)\\ \sigma_2^2 &= \frac{1}{2}(\sigma_{12}^2 + \sigma_{23}^2 - \sigma_{13}^2 - \sigma_{\text{link}}^2)\\ \sigma_3^2 &= \frac{1}{2}(\sigma_{13}^2 + \sigma_{23}^2 - \sigma_{12}^2 - \sigma_{\text{link}}^2).\end{aligned} \tag{12}$$

A similar approach was tested [9] in the 1980s and more recently in [10]. However, closures and closures variances were used only as stability and accuracy limitations due to GPS CV measurement noise over long baselines. They were not implemented as corrections in TCH computations of clocks under test.

III. Implementation of the Common-View Algorithm

Phase differences between clocks were measured using satellites of the GPS in CV. The oscillators are Symmetricom/HP 5071A cesium clocks. The three clocks belong to different institutions in Argentina, all of them separated by less than 20 km. The institutes are the Instituto Nacional de Tecnología Industrial (INTI), Observatorio Naval de Buenos Aires (ONBA), and the Instituto Geográfico Nacional (IGN).

The INTI uses its clock to contribute to Coordinated Universal Time (UTC), Rapid UTC (UTCr), and the Inter-American Metrology System (SIM) Time Scale, a multinational time scale [11]. The ONBA contributes to UTC and is in charge of the legal time in Argentina [12]. Finally, the IGN contributes to UTC and UTCr [13].

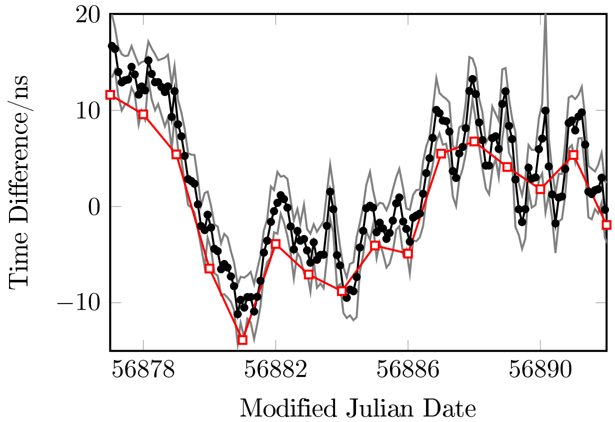

Fig. 1. Time differences between the INTI and the IGN calculated using UTCr and CV. The full black symbols are the results obtained by the CV algorithm, and the gray lines indicate the intervals of plus–minus one standard deviation. The open square symbols correspond to time differences extracted from the UTCr results. (For the sake of visibility, a linear trend was removed.)

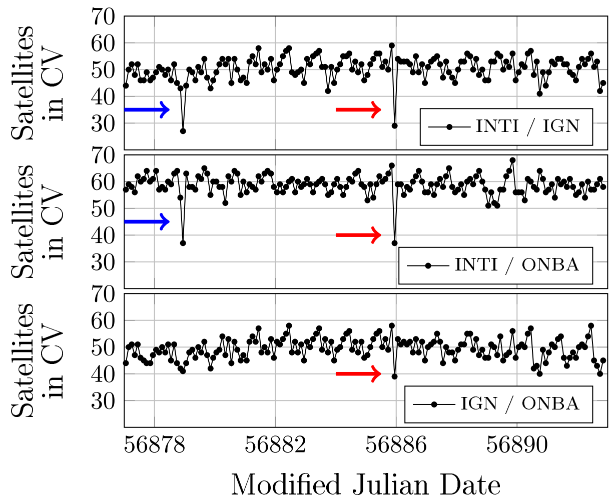

Fig. 2. Number of satellites in CV accumulated over 2.4 h for a 15-day period. Top: time differences between clocks maintained at the INTI and the IGN, computed for 2 months.

The GPS measurements of this work have been analyzed with an implementation of the CV algorithm described in reference [14].

A 3-MAD filter was applied to raw measurements data for the removal of outliers. This filtering procedure is performed with the aim of removing any outliers caused mainly by an unhealthy satellite. After this procedure, no further filters were implemented in the algorithm.

The mean values of time differences between the three laboratories were obtained each 2.4 h (i.e., a maximum of nine measurements of 16 min). Measurement data were collected over 2 months and missing data were reconstructed with interpolated values [15].

The good agreement between UTCr and CV values is shown in Fig.1. Here, time differences between the INTI and the IGN during 15 days are depicted.

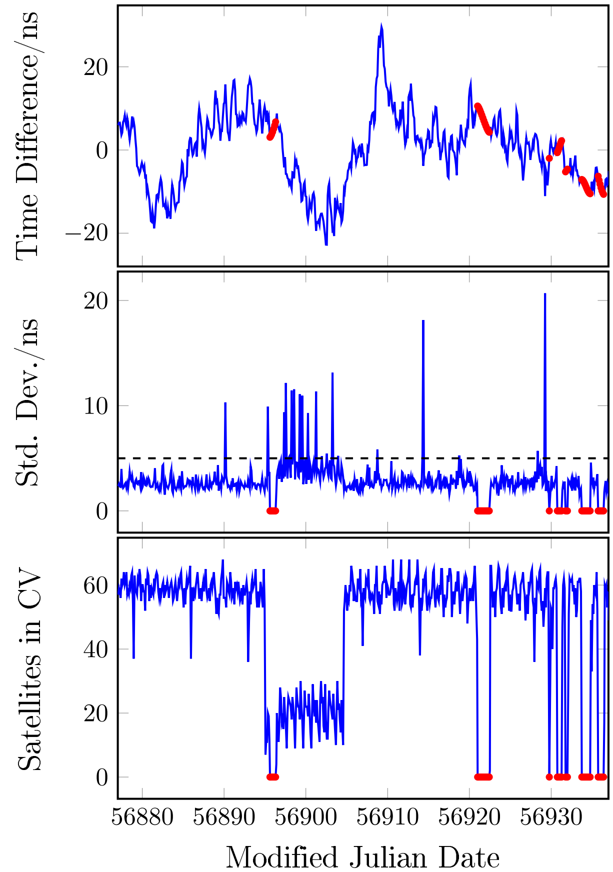

Fig. 3. Time differences, standard deviations, and satellites in CV for the INTI and IGN CV comparisons.

Fig. 2 shows the number of satellites in CV for the three different pairs of laboratories, during the same 15-day period as in Fig. 1. On days 56879 and 56886, there were short-term decreases in the number of CV measurements. In the first case (indicated by blue arrows in Fig. 2), the reason for this decrease was an issue in the receiver of the INTI, since there is no simultaneous anomaly in the IGN–ONBA number of measurements. On the contrary, during modified Julian date (MJD) 56886 (indicated by red arrows in Fig. 2), the decrease appears on the three pairs of comparisons, indicating an issue external to the laboratories' references and/or receivers. These short-term decreases in the number of satellites in CV did not have any impact on the means or the standard deviations of the time differences (see Fig. 1).

Long-term analysis of the CV measurements is shown in Fig. 3. After the removal of the linear trend in the phase differences (i.e., a frequency offset), the range of the values is 52 ns. The red segments in Fig. 3 indicate interpolated values due to the lack of CV measurements. The middle and bottom panels in Fig. 3 show the standard deviations of phase differences and the number of satellites in CV, respectively. Zero values are assigned to standard deviations during periods in which there were no CV measurements. Most of the time, the dispersions remained below 5 ns (see the dashed line in Fig. 3). Between MJDs 56895 and 56905, the number

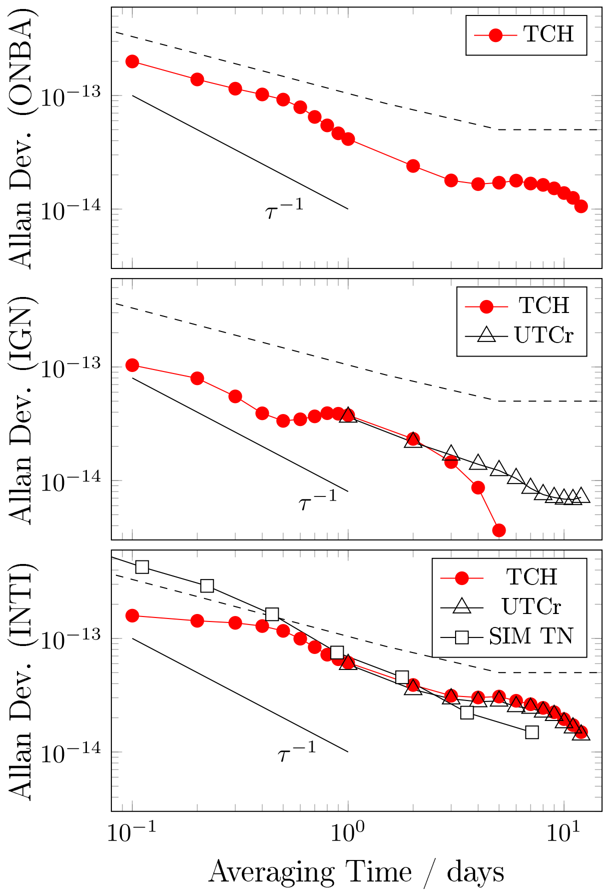

Fig. 4. Allan deviations of the three clocks analyzed. The dotted lines indicate the maximum instability specified by the manufacturer ($\tau^{-1/2}$ dependence for $\tau < 5$ days). An arbitrary τ^{-1} regime is plotted as reference.

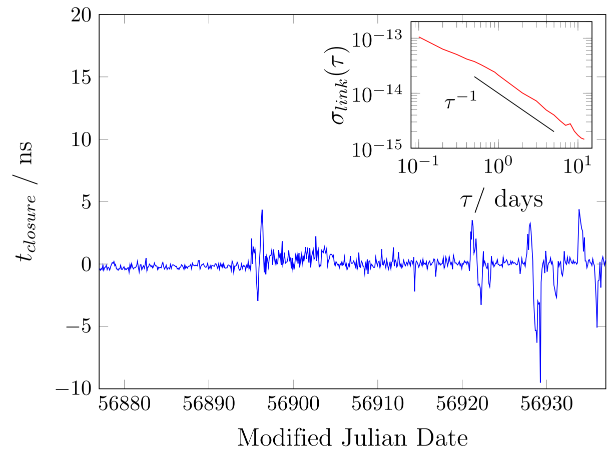

Fig. 5. Transfer noise in TCH via CV measurements: INTI–IGN–ONBA closure and its Allan deviation. An arbitrary τ^{-1} regime is plotted as reference.

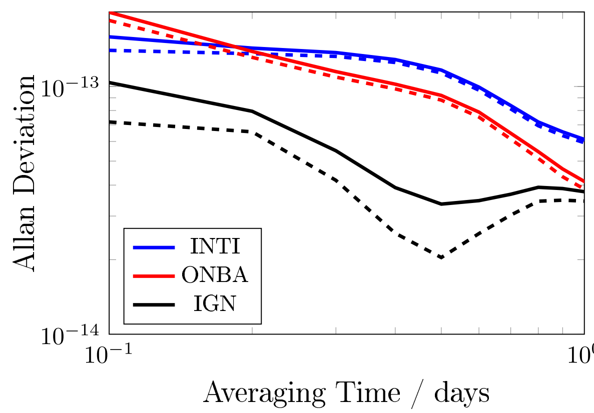

Fig. 6. Allan deviations obtained through TCH (solid lines) and corrected under the eTCH scheme (dashed lines).

of CV dropped below 30, causing an increase in the standard deviations. Nevertheless, no clear anomalies are present in the time differences (upper panel) during this 10-day period.

IV. Stability Evaluation

Fig. 4 shows the instabilities obtained for the three clocks. According to TCH estimates, all the clocks show stabilities according to their specifications (see the dotted lines in Fig. 4). As the ONBA contributes only to UTC, the only possible analysis for averaging times shorter than 7 days is the present work. At about $\tau = 0.5$ days, a small rise in the Allan deviation is obtained. This feature was already observed in the other CV analysis [10]. The likely source of this semidiurnal perturbation is the sensitivity of the antenna and cable with the outside temperature [16].

The variances obtained with TCH and through UTCr for the IGN are consistent between 1 and 3 days. Nonphysical results were obtained (negative Allan variance) for long averaging times in the IGN clock. This feature can appear in TCH estimates because of several reasons. One of them are correlations in the outputs of the clocks [17], [18]. In the present study, clocks are not placed in the same environment, so this seems not to be the limitation of the method in this case. Riley [3] suggests that negative variances arise when there is a clock with significative better stability than the other two. Dispersions in the values can also be an issue, in particular for the longer times, where the number of averaged phase differences is smaller.

Deviations obtained through TCH and UTCr for the INTI are in full agreement for $\tau = 1$ to 12 days. According to the SIM Time Network reports, results for $\tau < 0.5$ days slightly exceed the specification of the clock.

V. Estimation of Transfer Noise

Following the proposal of Section II, the obtained values for σ_{link} and t_{closure} are shown in Fig. 5. Closures are centered around 0 ns and only two measurements of the 600 exceeded 5 ns.

As expected, noise link levels are always below the variances of the individual clocks. For averaging times longer than 0.5 days, fluctuations in the closure behave like a τ^{-1} process, consistent with white noise phase modulation [19].

Fig. 6 shows the clock variances corrected under the eTCH scheme for $\tau = 0.1$ to 1 day. The proposed method improves the estimates up to about a 30% for $\tau = 0.1$ days on IGN's clock.

Given the steeper slope in comparison with the dependence of the three clock instabilities (Fig. 4), corrections for link noise are more significant at shorter averaging times than at larger ones.

VI. Conclusion

The possibility of making stability estimates combining the CV and the TCH techniques has been shown. The results are consistent with the ones obtained through the BIPMs *Circular T*.

This new approach allows the computation of absolute Allan variances with time averages down to 2.4 h, using GNSS signals. This may be of great importance for laboratories owning only one clock and need to evaluate their references at shorter intervals than UTC or UTCr reports.

An extended scheme for the TCH method has been proposed and tested. This enhanced version allowed the estimation of the noise added by the link in the comparison of clocks. The three clocks studied showed instabilities according to their specifications.

In the future, analyses over longer baselines should be carried on, in order to explore the robustness of the method.

Uncertainties in the variances obtained by the eTCH method must be evaluated, in particular the degrees of freedom remaining after the subtraction of the link noise.

References

[1] M. A. Lombardi, "A NIST disciplined oscillator: Delivering UTC(NIST) to the calibration laboratory," *J. Meas. Sci.*, vol. 5, no. 4, pp. 46–54, 2010.

[2] T. Gotoh, A. Kaneko, Y. Shibuya, and M. Imae, "GPS common view," *J. Nat. Inst. Inf. Commun. Technol.*, vol. 50, nos. 1–2, pp. 113–123, 2003.

[3] W. Riley, *Handbook of Frequency Stability Analysis*. Gaithersburg, MD, USA: Nat. Inst. Standards Technol., 2008.

[4] J. Levine, "Introduction to time and frequency metrology," *Rev. Sci. Instrum.*, vol. 70, no. 6, pp. 2567–2596, 1999.

[5] L. J. Gunn, P. G. Catlow, W. A. Al-Ashwal, J. G. Hartnett, A. Allison, and D. Abbott, "Simplified three-cornered-hat technique for frequency stability measurements," *IEEE Trans. Instrum. Meas.*, vol. 63, no. 4, pp. 889–895, Apr. 2014.

[6] C. Gao *et al.*, "The three corner hat measurement of three hydrogen masers in remote locations via fiber based frequency synchronization network," in *Proc. IEEE Eur. Freq. Time Forum (EFTF)*, Jun. 2014, pp. 259–261.

[7] M. A. Lombardi and A. N. Novick, "Remote time calibrations via the NIST time measurement and analysis service," *J. Meas. Sci.*, vol. 1, no. 4, pp. 50–59, 2006.

[8] P. Defraigne, W. Aerts, G. Cerretto, E. Cantoni, and J. M. Sleewaegen, "Calibration of galileo signals for time metrology," *IEEE Trans. Ultrason., Ferroelect., Freq. Control*, vol. 61, no. 12, pp. 1967–1975, Dec. 2014.

[9] D. W. Allan *et al.*, "Accuracy of international time and frequency comparisons via global positioning system satellites in common-view," *IEEE Trans. Instrum. Meas.*, vol. 34, no. 2, pp. 118–125, Jun. 1985.

[10] M. A. Weiss, G. Petit, and Z. Jiang, "A comparison of GPS common-view time transfer to all-in-view," in *Proc. IEEE Int. Freq. Control Symp. Expo.*, Aug. 2005, pp. 1–5.

[11] J. M. López-Romero, M. A. Lombardi, N. Diaz-Muñoz, and E. de Carlos-Lopez, "SIM time scale," *IEEE Trans. Instrum. Meas.*, vol. 62, no. 12, pp. 3343–3350, Dec. 2013.

[12] A. Cifuentes, C. Esperón, J. Osorio, J. Amenna, J. Salday, and G. Silva, "The public service of the official time at the observatorio naval buenos aires," in *Proc. Revista Mexicana Astronomia Astrofisica Conf. Ser.*, vol. 43. 2014, pp. 35–37.

[13] D. Gómez and S. Cimbaro, "The International Time Service of the National Geographic Institute (IGNA Laboratory) Argentina," in *Proc. Revista Mexicana Astronomia Astrofisica Conf. Ser.*, vol. 43. 2014, p. 38.

[14] R. Costa, D. Orgiazzi, V. Pettiti, I. Sesia, and P. Tavella, "GPS common view data processing algorithm," IEN, Torino, Italy, Tech. Rep. 677, 2004.

[15] I. Sesia and P. Tavella, "Estimating the Allan variance in the presence of long periods of missing data and outliers," *Metrologia*, vol. 45, no. 6, p. S134, 2008.

[16] G. Petit, C. Thomas, Z. Jiang, P. Uhrich, and F. Taris, "Use of GPS ASHTECH Z12T receivers for accurate time and frequency comparisons," *IEEE Trans. Ultrason., Ferroelect., Freq. Control*, vol. 46, no. 4, pp. 941–949, Jul. 1999.

[17] F. Vernotte, J. Delporte, and M. Brunet, "A re-revisited three cornered hat method for estimating clock instabilities," in *Proc. 18th Eur. Freq. Time Forum (EFTF)*, 2004, pp. 128–133.

[18] A. Premoli and P. Tavella, "A revisited three-cornered hat method for estimating frequency standard instability," *IEEE Trans. Instrum. Meas.*, vol. 42, no. 1, pp. 7–13, Feb. 1993.

[19] F. Riehle, *Frequency Standards: Basics and Applications*. Hoboken, NJ, USA: Wiley, 2006.

Diego Luna received the Ph.D. degree in physics from the University of Buenos Aires, Buenos Aires, Argentina, in 2014.

He joined the Instituto Nacional de Tecnología Industrial, Buenos Aires, in 2009. His current research interests include the time scales generation, time transfer, and remote calibrations.

Daniel Pérez joined the Instituto Nacional de Tecnología Industrial, Buenos Aires, Argentina, in 2003. He has participated actively in the contribution of Argentina to the SIM time and frequency network.

Alejandro Cifuentes received the Degree in astronomy from the University of La Plata, La Plata, Argentina.

Since 2006, he has been the Scientific Director with the Buenos Aires Naval Observatory. His current research interests include the astrometry and keeping legal time in Argentina.

Demián D. Gómez received the Ph.D. degree in geophysics from the University of Memphis, Memphis, TN, USA, in 2016.

He is currently a Post-Doctoral Researcher with Ohio State University, Columbus, OH, USA, and is also affiliated with the International Time Service, National Geographic Institute of Argentina. He is involved in GPS reference frame realizations as well as various geophysical projects.