STRATEGIES FOR BUILDING AN AC-DC TRANSFER SCALE

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Abstract: This paper presents the comparison between two strategies to build an AC-DC transfer scale. It is shown that an overdetermined scheme has some advantages over a simple one, though more time consuming. Three statistical tools are introduced to discard bad standards or measurements and to calculate uncertainty contributions. We compared the contributed uncertainties for both methods and results of their to an AC-DC current transfer scale.

1. INTRODUCTION

AC-DC current (or voltage) scales are built using the well known step-up-and-down procedures. At a medium level (typically 10 mA or 1 V), a set of well characterized thermal converters are taken as the basis of the system. For them, the AC-DC transfer difference is evaluated theoretically or determined in another lab. At other current (or voltage) levels, standards are calibrated against the standards of the neighboring range. The only assumption made is that the ac-dc transfer difference of each standard remains constant along its current (or voltage) range, from the reduced current (or voltage) at which it is calibrated against the neighboring standard to its higher rated current (or voltage).
A decision to be taken is the selection of the step-up strategy. Two basic approaches are possible: a direct one, where the largest possible jump is made with only one standard or an overdetermined scheme in which the jumps are made with more than one standard and the AC-DC transfer difference are overdetermined at each level. Fig 1 shows the two schemes for a current step-up that will be used further as examples.

2. THE DIRECT SCHEME

The AC-DC transfer difference of the standard being calibrated at level *i*, δ_i, is calculated as

$$\delta_i = \delta_{i-1} + \delta_d \tag{1}$$

where δ_{i-1} is the AC-DC transfer difference of the standard coming from the previous step, and δ_d is the measured difference between both standards. The standard uncertainty of δ_i is calculated as

$$u^2(\delta_i) = u^2(\delta_{i-1}) + u^2(\delta_A) + u^2(\delta_C) + u^2(\delta_S) + u^2(\delta_L) \tag{2}$$

where $u(\delta_{i-1})$ is the uncertainty of the standard coming from the previous step, $u(\delta_A)$ is the standard deviation of the measurements, $u(\delta_C)$ is the uncertainty contributed by the comparison system (i.e, linearity of nanovoltmeters, exponents of TCs), $u(\delta_S)$ is the uncertainty contributed by the measurement set-up (i.e, guarding, connectors) and $u(\delta_L)$ is the standard uncertainty contributed by the level dependence of the standard. In this method $u(\delta_S)$ and $u(\delta_L)$ are estimated from previous experience or indirectly evaluated, but, to some extent, subjectively.

3. THE OVERDETERMINED SCHEME

The overdetermined scheme is currently used at INTI. At 10 mA, five PTB thin-film multijunction thermal converters (PMJTCs), two of them together with shunts, are the basis of the system. The determination of the AC-DC transfer difference of the 5 PMJTCs depends on the frequency range. At audio frequencies (100 Hz < $f \leq$ 20 kHz) , the five PMJTC are compared among them and the mean value or the AC-DC transfer difference of three (1, 2 and 3) of the five PMJTCs are taken as zero. Hence, the following system of equations results

$$\begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 1 \\ -1 & 0 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \\ 0 \end{bmatrix} \tag{3}$$

where *a,b,c,d,e,f,* and *h* are the measured AC-DC transfer differences. The best solution [δ] is obtained using a modified least square method [1]. In this method the comparison statistic uncertainty and the lack of agreement of the fitting process are included in the uncertainty calculation. This lack of agreement is attributed to the changes of connections and positions of the standards in the measuring setup. At higher frequencies (20 kHz < $f \leq$ 1 MHz) the assigned value to TC-1 in a calibration against the PTB standards is used as a reference. To obtain the AC-DC transfer difference value of each standard, the equation system is solved using the least square method. Below 100 Hz, a different strategy is used out of the scope of this paper [2].

To step-up, we used two standards to jump from one range to the other. At the highest range of the leap, one of these standards is at its rated power and the other one is at a quarter of it. The redundancy is necessary for the statistical tools that will be introduced. At each current level a system of equations is obtained. For instance, at 50 mA we get,

$$\begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \delta_4 \\ \delta_6 \\ \delta_7 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ \delta_4^{'} \\ \delta_6^{'} \end{bmatrix} \quad (4)$$

i.e $$[\mathbf{A}] \cdot [\delta] = [\mathbf{B}] \quad (5)$$

where δ'_4 and δ'_6 are the values obtained for these standards in the previous steps. Using the least square method

$$\delta' = \left(\mathrm{A}^T \cdot \mathrm{A}\right)^{-1} \cdot \mathrm{A}^T \cdot \mathrm{B} = \mathrm{C} \cdot \mathrm{B} \quad (6)$$

The residual vector $B - A \cdot \delta'$ represents the lack of fit of the model. Its associated uncertainty can be quantified from the residual sum of squares and the residual variance

$$SS = \left\| \mathrm{B} - \mathrm{A} \cdot \delta' \right\|; \quad \hat{\sigma}^2_{step} = SS / df \quad (7)$$

where df, the number of degrees of freedom used to estimate the residual variance, is calculated as the number of rows minus the number of columns of A. The uncertainties of δ' are calculated as the square root of the diagonal terms of the covariance matrix

$$\mathrm{cov}(\delta') = \mathbf{C} \cdot \mathrm{cov}(\mathbf{B}) \cdot \mathbf{C}^T \quad (8)$$

and cov(B) is the covariance matrix of B. The diagonal terms of cov(B) are:

$$\mathrm{var}(b_{ii}) = u^2(\delta_{Ai}) + u^2(\delta_{Ci}) + u^2(\delta_{Mi}) \quad i=a,b,c \quad (9)$$

$$\mathrm{var}\ (ii) = u^2(\delta_{ip}) \qquad i=d,e \quad (10)$$

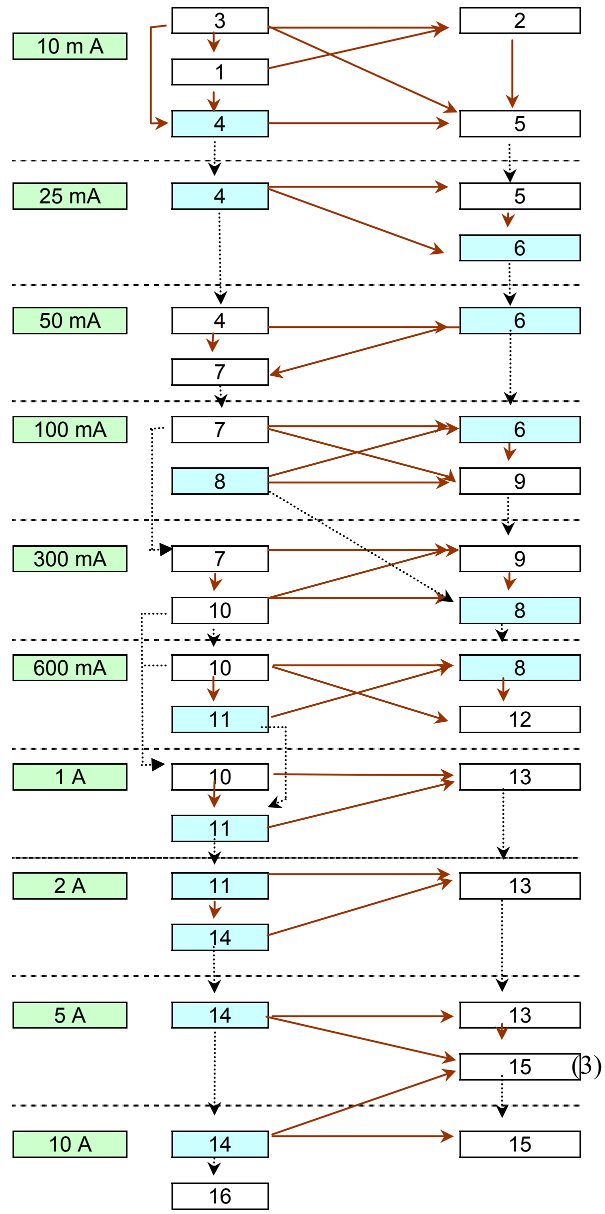

Fig 1 - Overdetermined AC-DC current step-up. In light blue is also shown one possible direct scheme with only one TC jumping between levels.

where $u(\delta_A)$ is the Type A standard uncertainty associated to the repeatability of each bilateral comparison, $u(\delta_C)$ is the Type B standard uncertainty associated to the comparison system, $u(\delta_M)$ is the standard uncertainty of the scheme comparison, which, in our approach, can be estimated as the residual standard deviation of the least square fit, that is,

$$u_M(i) = \sqrt{\hat{\sigma}^2_{step}} \quad (11)$$

and $u(\delta_p)$ is the standard uncertainty of the AC-DC transfer difference of the standards coming from the previous step [2]. The standard uncertainty of δ_i is

$$u\left(\delta_i^{''}\right) = \sqrt{\operatorname{cov}(\delta_{ii}^{'})} \qquad (12)$$

Both schemes require thermal converters with level independent ac-dc differences and good stability of their ac-dc difference. The overdetermined scheme allows for the use of statistical tests to check these requirements quantitatively and objectively.

3.1. Statistical Tests

3.1.1. Testing the Level Dependence.

We propose a way to test whether the AC-DC transfer difference of a standard does not depend on the current level. This test should be applied to all the pairs of standards used to go up (or down) from one range to another. Let us suppose that two of these transfers, A and B, are used to go from a current level 1 to another level 2, and let us call A1 and B1 their values in level 1, and A2 and B2 the corresponding values in level 2 (Fig.2). Both standards have been compared *n* times at both levels, and the averages and standard deviations of the measured differences have been calculated.

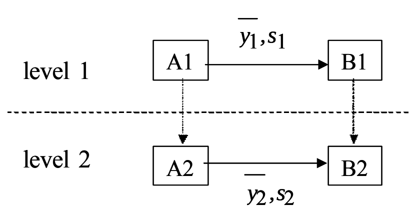

Fig.2. Step with two transfers as reference

If both standards are equally affected by the level change, the averages will be similar at both levels. If not so, we could conclude that one of them is more affected than the other one. Both PMJTCs are of similar design and technology but are used at quite different power, that is, different internal temperature. Thus, if it exists a change between $\overline{y_1}$ and $\overline{y_2}$ it can be assigned to the most powered PMJTC. To test if the difference between $\overline{y_1}$ and $\overline{y_2}$ is statistically significant, a simple two-sample *t*-test for mean differences [3] is applied, based on the statistic

$$T = \left(\overline{y_1} - \overline{y_2}\right) \Big/ \sqrt{\frac{s_1^{\,2} + s_2^{\,2}}{n}} \qquad (13)$$

A value of $|T|$ greater than a critical value $t_{2n-2;\alpha/2}$ (which depends on a previously stated type of risk α) leads to conclude that the difference between the averages is statistically significant and, therefore, the level dependence must be considered an uncertainty component, assigned to the most powered standard. Otherwise, it can be assumed that the difference is negligible or attributed to random errors, which are contemplated in the least square calculation. For example, with the data from step 10 mA to 25 mA at 100 kHz, with n=12, $\overline{y_1} = 10{,}91$, $\overline{y_2} = 13{,}32$, s_1=0,79, s_2=0,22, we obtain $|T|$=2,939. If we use an α = 0,05, $t_{2n-2\alpha/2}$ =2,074. Thus, $|T|$> $t_{2n-2\alpha/2}$, and we conclude that the difference between the averages is statistically significant. Therefore, the uncertainty caused by this factor must be considered. Its standard uncertainty is estimated as

$$u_{ld} = \frac{\left|\bar{y}_1 - \bar{y}_2\right|}{\sqrt{3}} \qquad (14)$$

and incorporated to the uncertainty of the most powered standard.

3.1.2. Testing the Consistency between Pairs of Standards.

To verify that the values assigned to both reference standards A and B at the same step are consistent, we propose to compare the results obtained by solving the step twice, according to the following procedure:

- First, the step is solved, considering both standards providing a link condition to the previous step. Let us call δ_{AB} the output vector of the step
- Then, one of the link conditions is eliminated from the model (deleting one of the two last lines in the design matrix A). Therefore, other values will be obtained for all the transfers, δ_B

Finally, both estimations are compared by means of the parameter E_n [4],

$$E_n(i) = \frac{\delta_{AB}(i) - \delta_B(i)}{u(\delta_{AB}(i) - \delta_B(i))} \qquad (15)$$

The standard uncertainty in the denominator must be calculated suppressing all the correlations between δ_{AB} and δ_{AB}. Values of $E_n(i)$ greater than 2 for any *i* express lack of consistence. . For example, at 50 mA 100 kHz, we obtain,

δ_{AB}={14.07; 19.15; 11.18}

for the transfers PMJTC-4+SH-1, PMJTC-1+SH-3, PMJTC-3+SH-4, respectively.

δ_B={15.65; 21.73; 13.26}

$|\delta_{AB} - \delta_B|$ = {1.57; 2.58; 2.086}

$u(\delta_{AB} - \delta_B)$ = {10.15; 10.14; 10.14}

Therefore, E_n={0.15; 0.25; 0.21}

As E_n is always smaller that 2, we conclude that there is consistency between the two reference standards at this step.

3.1.3. Testing the Stability of a Standard

A statistical method to test the inner consistency of each step is proposed. If a transfer is not stable enough along the time when the measurements are performed, the least square fit will be poor and the residual standard deviation (7) will be too high. So, to test the hypothesis of consistency, it can be compared with the fit achieved from a reduced model. If one of the transfers is suspected of being unstable, it is discarded from the scheme. A reduced matrix A_r is obtained from A, by eliminating the column corresponding to the discarded standard, and all the rows related to the measurements in which this standard was involved. Also, a reduced vector of observations B_r is obtained from B, and new estimations for the non-discarded transfers can be calculated.

Following the same procedure than for the full model, the reduced sum of squares SS_r, the reduced degrees of freedom df_r and the reduced residual variance $\hat{\sigma}_r^2$ are obtained. Then, an F-statistic can be calculated as

$$F = \frac{(SS - SS_r)/(df - df_r)}{SS_r / df_r} \qquad (16)$$

It can be shown that $SS - SS_r$ and SS_r are distributed according to χ^2 distributions with $df - dfr$ and dfr degrees of freedom, respectively, and that both quantities are statistically independent [5]. Thus, F is distributed according to a *Fisher-Snedecor* distribution with $df\text{-}df_r$ degrees of freedom in the numerator and df_r degrees of freedom in the denominator [5].

A type one risk α (the risk of detecting a non-existing instability) is previously stated. Thus, if the calculated value of F is greater than the tabulated critical value $f_{df-dfr,dfr,\alpha}$ we can conclude that the model consistency is significantly weaker for the full model than for the reduced one. Then, the lack of stability of the separated transfer can be considered significant. The power of the F-test –that is, the probability of detecting an actual lack of consistence- was evaluated by Monte Carlo simulations. As an example, Fig. 4 shows the 10 mA step with 5 transfers, where transfer 4 was evaluated as possibly unstable. Simulated results of measurements are obtained assigning random numbers to each pair-comparison in the step. Such random numbers are generated from gaussian distributions with a common mean value 0 and a common standard deviation σ_{step}, which is the combination of the Type A sources of uncertainty associated to the step (lack of fit and repeatability). During the simulation, one of the comparisons in which the suspected transfer participates was contaminated, adding increasing constant biases between 0 and $5\cdot\sigma_{step}$. Then, for each value of contamination, the simulation was repeated M=5000 times, recording when the contamination was detected by the F-test. Fig. 3 depicts that the F-test power is not good. For instance, the test for α=0.10 detects a $3\cdot\sigma$ contamination with a probability close to 26%.

To increase the power of the method, we introduce a modification of the test based on the Monte Carlo simulation of the measurements.

Each one of the comparisons presented in the step is repeated by the generation of N random numbers with gaussian distributions centered in the average of an actual measurement. Those generations are performed with a common standard deviation. Simulated versions of the F statistics $F_1 \ldots F_N$ are computed by means of the same procedure that for the F-test. These copies of F could be used for statistical calculations. However, as the mathematical properties of F are hard to work with, we compute $\log(F_i)$ which has a probability distribution not so far from the gaussian one. So, the following statistics can be obtained

$$T = \frac{\overline{\log(F)} - \mu_{\log(F)}}{s(\log(F))/\sqrt{N}} \qquad (17)$$

where $s(\log(F))$ is the sample standard deviation of $\log(F_1) \ldots \log(F_N)$, and $\mu_{\log(F)}$ is the theoretical expected value of $\log(F_i)$, which can be calculated following the pdf of the F distribution [3], as follows

$$\mu_{\log(F)} = \frac{\Gamma\left(\frac{n+m}{2}\right) n^{n/2} m^{m/2}}{\Gamma(n/2)\,\Gamma(m/2)} \int_0^\infty \frac{\log(x)\cdot x^{n/2-1}}{(m+n\,x)^{(m+n)/2}} dx = 0 \qquad (18)$$

being $n = df - dfr$ and $m = dfr$. For instance, for the step as in Fig. 4, the test for transfer number 4 gives $df = 4$, $dfr = 2$, $n = m = 2$, and

$$\mu_{\log(F)} = 2\int_0^\infty \frac{\log(x)}{(1+x)^2} dx = 0 \qquad (19)$$

The distribution of T can be approximated by a t one, with $N-1$ degrees of freedom. So, the condition to conclude instability or lack of consistence in the step is

$$T > t_{N-1,\alpha} \qquad (20)$$

The power of the T-test was evaluated for the same case and in a similar way than for the *F*-test. The results for 100000 simulations are shown in Fig 3.

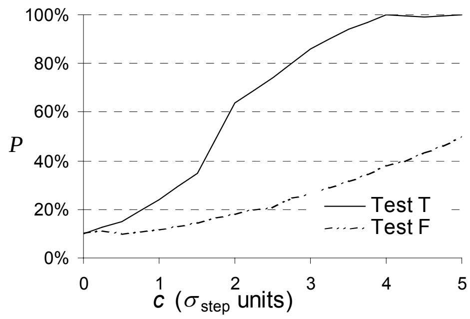

Fig. 3 Power of the F anf T-tests. P is the percentage of detection and c is the contamination in σ units.

Note that, for each contamination, M·N simulations were needed: M values simulating the measurement results must be generated, and, for each one of these, N simulated F_i must be obtained.

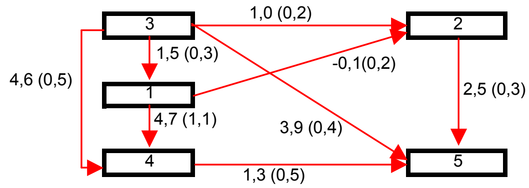

Fig. 4 – Comparison of 5 standards at 10 mA, 10 kHz. The values near the arrows are the measured values with their standard deviation in brackets

Fig. 4 shows the results of the measurements at 10 A at 1 kHz. If we apply eq. (13) to each standard suspected of being unstable we get T_1=0,55, T_2=0,25, T_3=0,29, T_4=2,2, T_5=0,88. If we choose a type one of risk of 10%, we get critical $t_{4,0.1}$=1,64 [3]. As T_4 > 1.64, we conclude that the transfer 4 is unstable and should be replaced.

4. RESULTS

The uncertainty was evaluated for the two schemes shown in Fig. 1. To show the contribution of the step-up clearly, the uncertainty of the basic standards at 10 mA was taken as zero. As an example, Table I depicts the contribution for the direct scheme at 5 A.

The contributions for the overdetermined scheme are calculated from eq. (12). Table II shows the calculated uncertainties.

Table I Examples of uncertainties components for the direct method at 5 A in $\mu A \cdot A^{-1}$

Component	f = 1 kHz	f = 20 kHz	f =100 kHz
$u(\delta_A)$	0,2	0,2	0,5
$u(\delta_C)$	0,2	0,2	0,5
$u(\delta_S)$	1	1	2
$u(\delta_L)$	1	2	4

Table II Standard uncertainties calculated for the direct (D) and the overdetermined (O) schemes in $\mu A \cdot A^{-1}$

Standards	f = 1 kHz		f = 20 kHz		f =100 kHz	
	D	O	D	O	D	O
0,10 A	0,9	0,6	1,3	1,0	3,2	2,3
0,6 A	1,2	0,7	1,8	1,1	3,9	2,7
5 A	1,8	0,9	2,7	2,0	7,3	3,0

5. CONCLUSIONS

The use of an overdetermined scheme to step up allows for the use of statistical tests to assess the quality of a step-up scheme. Unstable or level dependant standards can be discarded with a base on objective numbers. Therefore, the contribution of the step-up to the uncertainty can be reduced. Besides, the uncertainty components can be calculated from the measurements. The direct method needs less measurements, but some uncertainty components must be estimated from the previous experience.

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