CENAM
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Centro Nacional de Metrología

SIM Regional Key Comparison SIM.L-K1.2007

Calibration of Gauge Blocks by Optical Interferometry

FINAL REPORT

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Contents

1. Introduction

The Mutual Recognition Arrangement (MRA) of the *Conférence Internationale des Poids et Mesures* (*CIPM*) signed by the National Metrology Institutes (NMI) of different nations provides mutual recognition among the NMI of their national standards and their calibration services. A database has been set up by the *Bureau Interantional des Poids et Mesures* (*BIPM*) at its website where the Calibration and Measurement Capabilities (CMC) of each NMI are posted. To support the CMC claims of the NMI, the MRA requires, among other things, that they participate, on a regular basis, in Key Comparisons (KC) that test key measuring techniques. This would prove their technical competence, that they can provide this calibration service with the claimed uncertainty of the corresponding CMC and that they have metrological equivalence with the other signatory NMI that provide the same calibration service.

KC should take place at the highest level amongst the members of the corresponding Consultative Committee (CC), in this case the Consultative Committee for Length (CCL). Similar regional KC should also be organized in every region with at least a few NMI from the region participating in the regional comparison as well as in the CCL KC.

The CIPM has also instructed the different CC to identify key techniques in order to define KC. The calibration of Gauge Blocks (GB) by optical inteferometry has been identified as a key measuring technique by the Consultative Committee for Length (CCL). In one hand it requires good technical expertise and skills, the use of sophisticated equipment and stringent laboratory conditions; but in the other hand it is an unavoidable step in the dissemination of the length unit and therefore it is of paramount importance. These KC have been designated as K1 comparisons.

Both levels of comparisons should be organized regularly in time at a frequency established by each CC. The present comparison, SIM.L-K1.2007, is the second K1 comparison organized by SIM region since the signature of the MRA. It is intended to support and maintain the posted CMC of the NMI of the Americas that offer GB calibration by optical interferometry on the database, and, eventually, any other calibration services that stems out of this key technique.

The mesurand is the central length of the GB as defined in [1]. Additionally, for those laboratories willing to participate, a Pilot Study on the Optical Phase Change Correction on the Reflection of Light has been organized along with SIM.L-K1.2007 using the same GB and a set of platens that were circulated along with the GB. The results will be part of a separate Pilot Study report and are not part of this comparison.

The optical interferometry measurement of the GB is a first stage of the circulation of these GB. A second stage has been organized to measure them by mechanical comparison. The circulation of this second stage has just ended and, therefore, we can now disclose the results contained in this report.

The comparison had nine participants, five from the Americas, and four invited ones from other regions. The circulation took more than two years, from November 2007 until March 2010. The exercise was quite delayed as the allocated time periods could not be respected at several points.

2. Participants

A total of nine NMI participated in this comparison that circulated 14 GB of different lengths and materials. The following table lists the information on the participating NMI.

Contact	NMI	Information
Joaquín Rodríguez González, Emilio Prieto Esteban	**CEM,** Centro Español de Metrología Alfar, 2. Tres Cantos 28760 Madrid, España	Tel. +34 91 8074 796 / 801 Fax +34 91 8074 807 e-mail: jrgonzalez@cem.minetur.es ; eprieto@cem.minetur.es
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Table 1. List of participants in comparison SIM.L-K1.2007.

[1] Formerly: Institute for National Measurement Standards (INMS); update contact: Pierre Dube pierre.dube@nrc-cnrc.gc.ca

3. Circulation Schedule

The circulation of the artifacts was very delayed mainly due to customs clearance delays in several countries. A circulation time of 14 months was initially scheduled and it took 30 months, more than the double of the initial scheduled time. Table 2 shows the actual dates of reception and shipment of the artifacts by the participants as well as the date of reception of the participant's results by the pilot laboratory.

NMI	Dates		Reception of Results
	Reception	Shipment	
CENAM (Pilot)		2007-11-01	2007-11-10
CEM	2007-11-23	2008-01-14	2008-09-01
NPLI	2008-03-02	2008-06-09	2009-08-07
CMI	2008-06-16	2008-08-05	2008-10-17
NMISA	2008-09-10	2008-11-27	Did not send-in results
NRC-INMS	2008-12-04	2009-04-17	2009-08-25
NIST	2009-04-20	2009-07-15	2010-03-02
INMETRO	2009-08-11	2009-09-08	2009-09-21
INTI	2009-11-13	2010-01-11	2010-01-11
CENAM (Pilot)	2010-03-02		2010-04-25

Table 2. SIM.L-K1.2007 dates of reception and shipment of artifacts and reception of results by the pilot lab.

4. Comparison Artifacts

A total of 14 grade K (according to [1]) rectangular GB were selected for the exercise. Seven steel GB and seven ceramics GB covering the range of short GB (from 0.5 mm to 100 mm). The specifications on the GB are shown in **tables 3** and **4**. The associated Coefficients of Thermal Expansion (CET) shown in the tables are those quoted by the manufacturers.

Nominal Length (mm)	Serial Number	Coefficient of Thermal Expansion (10^{-6} K^{-1})	Manufacturer
1.000 5	010223	10.9 ± 1	Mitutoyo
5	000482	10.9 ± 1	Mitutoyo
7	010764	10.9 ± 1	Mitutoyo
10	001329	10.9 ± 1	Mitutoyo
50	012254	10.9 ± 1	Mitutoyo
75	010630	10.9 ± 1	Mitutoyo
100	010850	10.9 ± 1	Mitutoyo

Table 3. Steel Gauge Blocks.

Nominal Length (mm)	Serial Number	Coefficient of Thermal Expansion (10^{-6} K^{-1})	Manufacturer
1.000 5	000288	9.3 ± 1	Mitutoyo
5	051836	9.3 ± 1	Mitutoyo
7	010323	9.3 ± 1	Mitutoyo
10	052351	9.3 ± 1	Mitutoyo
50	011002	9.3 ± 1	Mitutoyo
75	010370	9.3 ± 1	Mitutoyo
100	010773	9.3 ± 1	Mitutoyo

Table 4. Ceramics Gauge Blocks.

5. Measurement Protocol

Detailed Measurement Instructions were included in the Comparison Protocol. The GB were supposed to be measured wrung to the platens or optical flats that the participant laboratories currently use to offer their gauge block calibration service.

Gauge block calibration by optical interferometry should be performed with the GB in vertical position wrung to a platen as indicated in [1]. The gauge block central length, $l_{c,}$ is the perpendicular distance between the central point of the free measurement surface of the gauge block and the surface where it is wrung.

The values asked to be reported in the protocol were the deviations from nominal length (l_n) determined at the center for each measuring face “A” and “B”, $e_{cX} = l_c - l_n$, (where X = “A” or “B”); the average of both values, e_{avg}; the so called phase change correction, Δl_ϕ; and the corrected average deviation after applying the phase change correction, e_c.

The method most commonly used to determine the phase change correction, Δl_ϕ is the stack method and it is described in **Annex E**.

6. Measuring Instruments

All participant laboratories measured the GB by optical absolute interferometry applying the method of exact fractions. The systems used, traceability, light sources and laboratory temperature variations of the participants are listed in **table 5**.

7. State and Behavior of Artifacts

7. 1 State of the Artifacts upon Reception

The participants were to inspect the state of the artifacts upon reception and inform the pilot according to the protocol. Although the selected GB were not new, they were in good conditions for measurement. A few of the steel GB suffered some damage after the circulation, but the results obtained in the comparison prove that the damages did not hamper or alter the measurements; and the pilot laboratory was able to wring them all to a platen at the end of the circulation. **Figures 1** through **4** show the physical conditions of some of the damaged GB upon reception by the pilot laboratory at the end of the circulation.

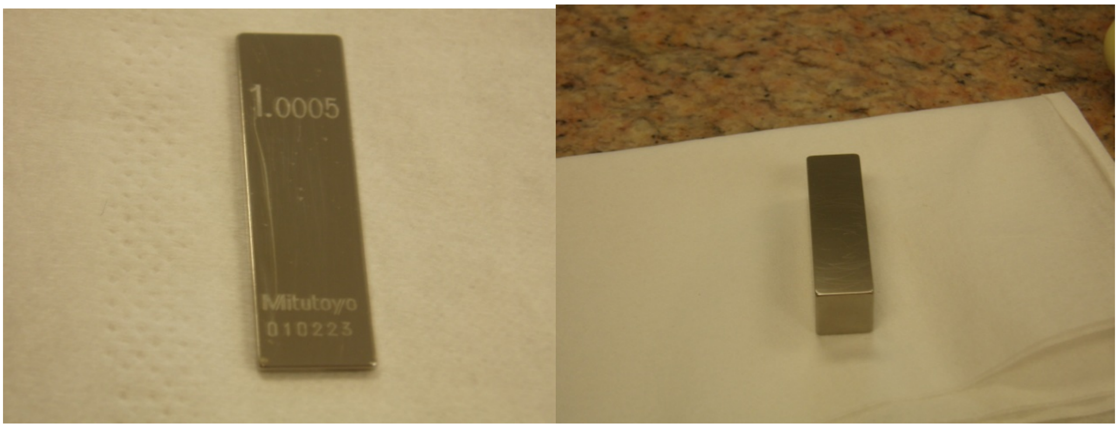
NMI	Manufacturer and Type of Interferometer	Light sources and wavelengths used	Traceability	Temperature variation range during measurements (°C)
CEM	NPL-TESA, Twyman-Green	He-Ne ~633 nm TESA laser, He-Ne ~543 nm TESA laser,	To the Spanish realization of the metre: A ~633 nm Iodine-stabilized laser; and to a UK ~543 nm Iodine-stabilized laser.	19.891 – 20.156
CENAM	NPL-TESA, Twyman-Green	He-Ne ~633 nm TESA laser, He-Ne ~543 nm TESA laser,	To the Mexican realization of the metre: A ~633 nm Iodine-stabilized laser (CNM-PNM-2)	19.95 – 20.15
CMI	NPL-TESA, Twyman-Green	He-Ne ~633 nm TESA laser, He-Ne ~543 nm TESA laser,	To the Czech National Standard of Length (He-Ne/I2 633nm, He-Ne/I2 543.5nm, fs comb)	19.640 – 20.198
INMETRO	Jena Zeiss Michelson/Twyman-Green	Double cathode ^{114}Cd spectral lamp	To SI standards of INMETRO	19.92 – 20.36
INTI	NPL-TESA, Twyman-Green	He-Ne ~633 nm TESA laser, He-Ne ~543 nm TESA laser,	To SI standards of INTI	Not specified
NIST	Hilger	He-NE ~633 nm Spectra Physics laser,	NIST maintained Iodine-Stabilized Laser	20.028 – 20.035
NMISA	-----	------	-------	-----
NPLI	NPL-TESA, Twyman-Green	He-Ne ~633 nm TESA laser, He-Ne ~543 nm TESA laser,	Not specified	19.6 – 20.3
NRC-INMS	NRC-INMS own design, Twyman-Green	He-Ne ~633 nm COHERENT laser, He-Ne ~612 nm TESA laser, He-Ne ~543 nm TESA laser, He –Ne ~1 152 nm laser,	The ~633 nm is traceable to an Iodine-Stabilized laser. The other wavelengths are traceable to the primary time and frequency standard of Canada.	19.986 – 20.019 for steel 19.971 – 20.020 for ceramics

Table 5. GB interferometers, laser sources, traceability and temperature variation of the participant laboratories.

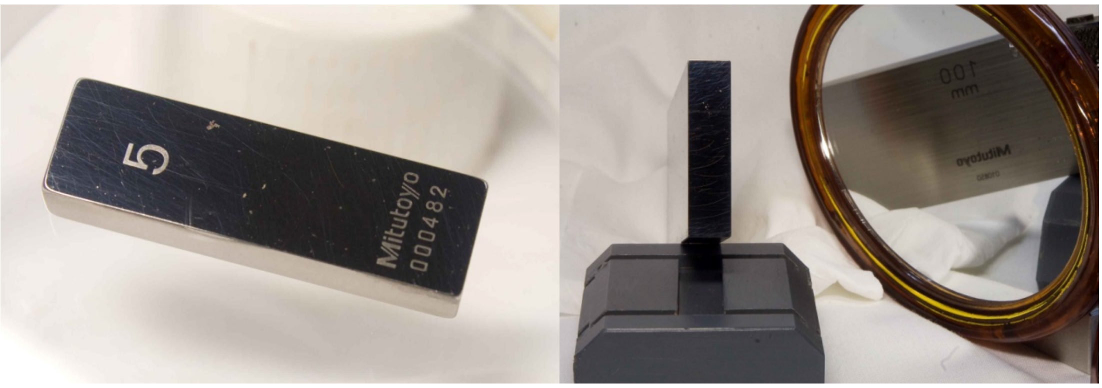
Problems on some GB were reported by two participant NMI:

- ***CEM* from Spain**.
  - The Ceramics 100 mm GB (serial no. 010773) presented wringing problems, but they were able to submit measurement results.
  - The case where the GB were packed was received with damages that were apparently suffered during transportation between Mexico and Spain.

- ***INMETRO* from Brazil.**
  - Reports having had difficulties in wringing some GB due to damage on the measuring faces, but they were also able to provide measurement results.



Figures 1 and 2. Physical condition of the 1.000 5 mm GB and the 10 mm GB after circulation. Notice the scratches and spots on the measuring faces.



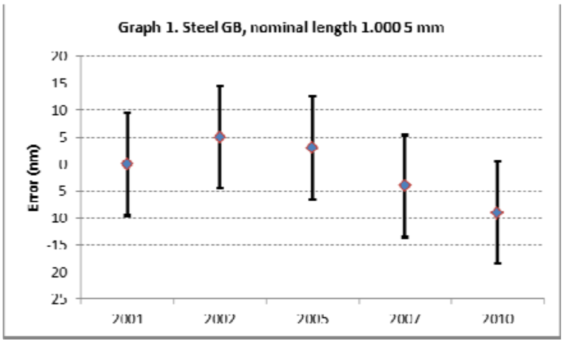
Figures 3 and 4. Physical condition of the 5 mm GB and the 100 mm GB after circulation. Notice the scratches and spots on the measuring face of the first one; and indentations and scratches in second one.

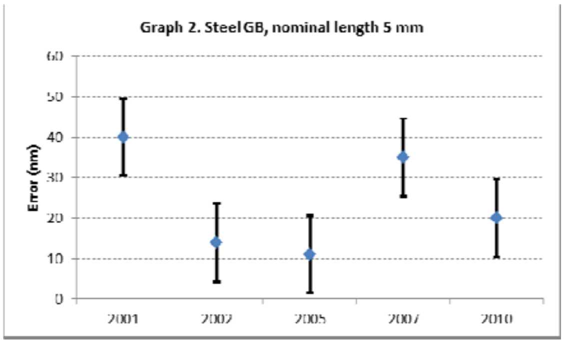
7. 2 Stability of the Standards

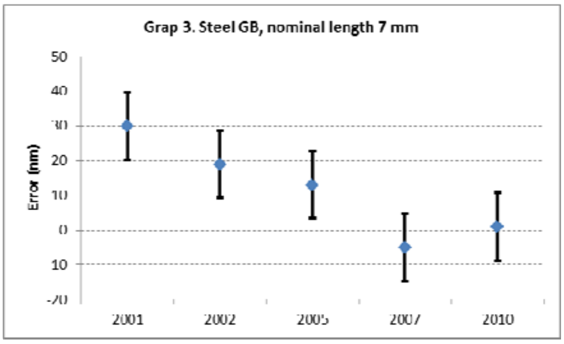
The GB were measured several times by the pilot laboratory to verify their stability: when they were purchased (2002), two years before starting the comparison (Nov. 2005), before circulating them (Nov. 2007) and at the end of the circulation (April 2010). Table 6 shows the deviations from nominal length determined at these different occasions for the steel GB, including the stated values on the certificates of the manufacturer. **Graphs 1** through **7** show these values for each GB along with the corresponding standard uncertainty bars.

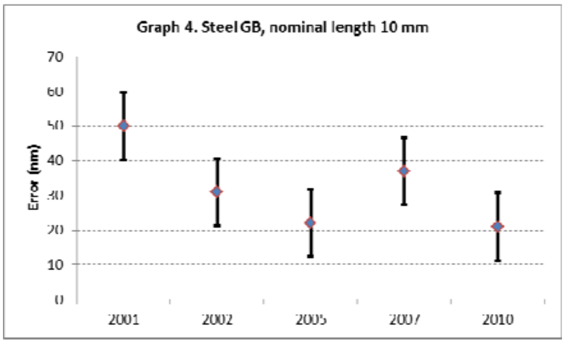
Serial Number	**Nominal Length** (mm)	**Deviation from nominal value** (nm)				
		Manufacturer certificate 2001	**2002**	**2005**	**2007**	**2010**
010223	1.000 5	0	5	3	-4	-9
000482	5	40	14	11	35	20
010764	7	30	19	13	-5	1
001329	10	50	31	22	37	21
012254	50	60	46	3	7	-3
010630	75	-50	-54	-104	-100	-107
010850	100	20	18	-50	-51	-64

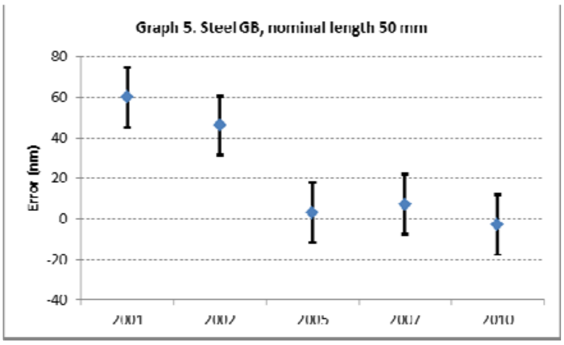
Table 6. Pilot Laboratory measured values of the steel GB at different occasions.

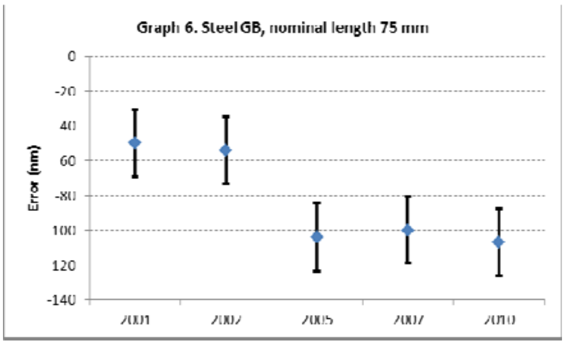












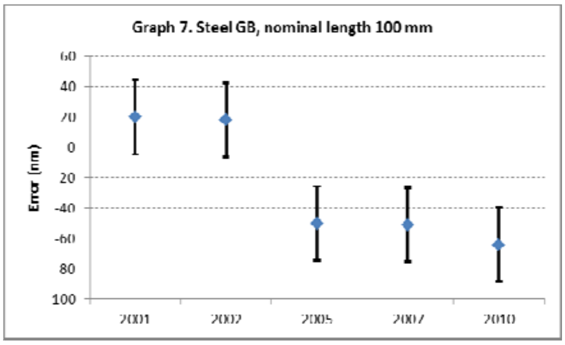
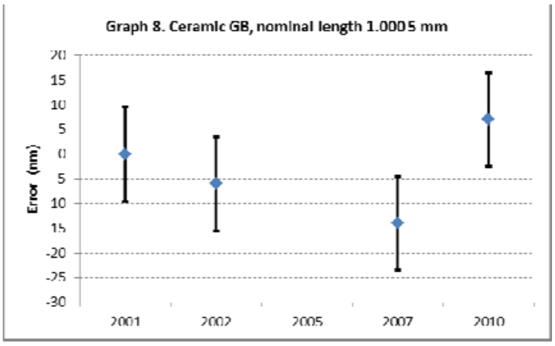
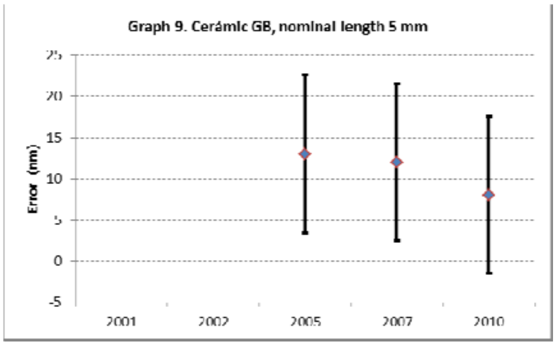


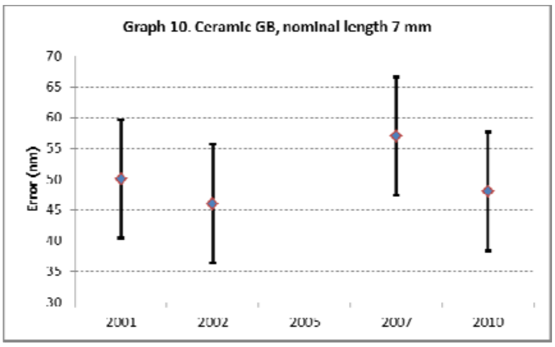
Table 7 shows the deviations from nominal length determined at these different occasions for the ceramics GB, including the stated values on the certificates of the manufacturer. **Graphs 8** through **14** show these values for each GB with its standard uncertainty bars.

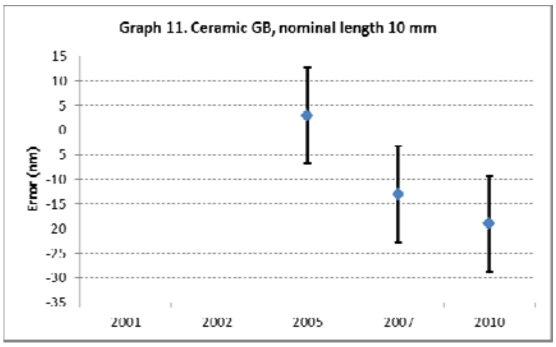
Serial Number	**Nominal Length** (mm)	**Deviation from nominal value** (nm)				
		Manufacturer certificate 2001	**2002**	**Manufacturer certificate 2005**	**2007**	**2010**
000288	1.000 5	0	-6	----	-14	7
051836	5	----	----	13	12	8
010323	7	50	46	----	57	48
052351	10	----	----	3	-13	-19
011002	50	90	95	----	139	117
010370	75	100	110	----	118	124
010773	100	-60	-42	----	-34	-36

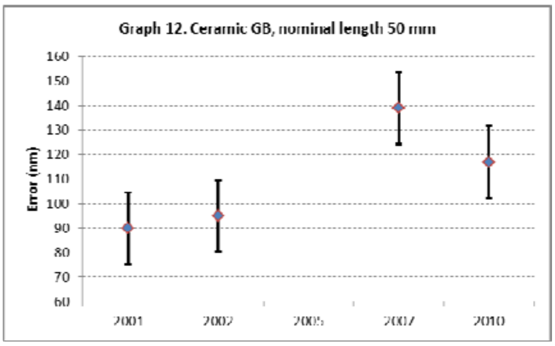
Table 7. Pilot Laboratory measured values of the ceramics GB in different occasions.

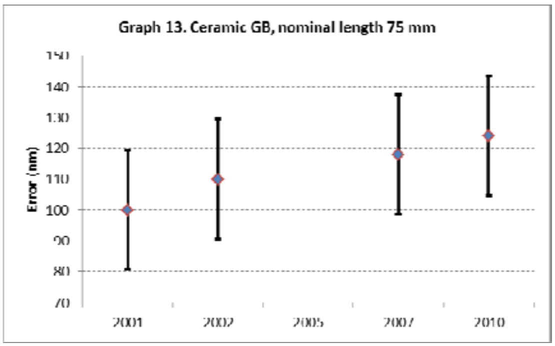


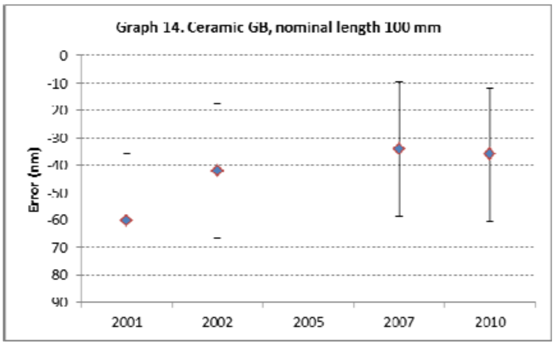












8. Measurement Results of Participants

All laboratories sent their results by e-mail. NRC-INMS sent them by parcel service as well. All information was received on the specified formats from appendices A, B, C, D and E of the Technical Protocol.

8.1 Measurement of the Central Length

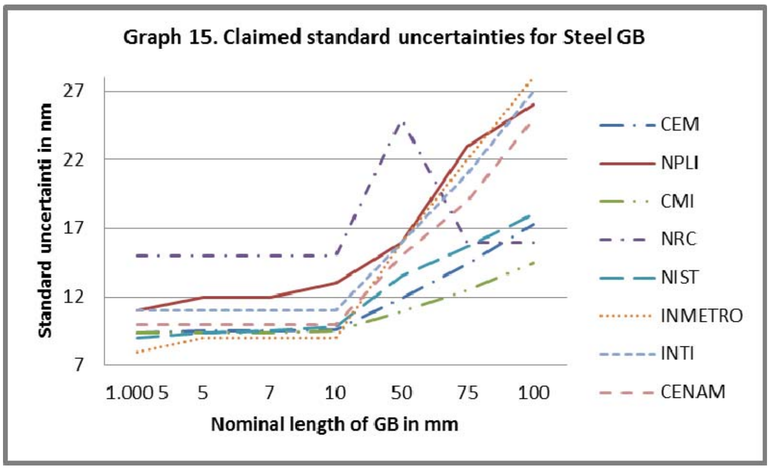
Tables 8 and **9** and **graphs 16** through **22**, show the deviations of the central length with respect to nominal values and the claimed standard measurement uncertainties of each participant for the seven steel GB; and **graph 15** show the claimed standard uncertainties of the participants.

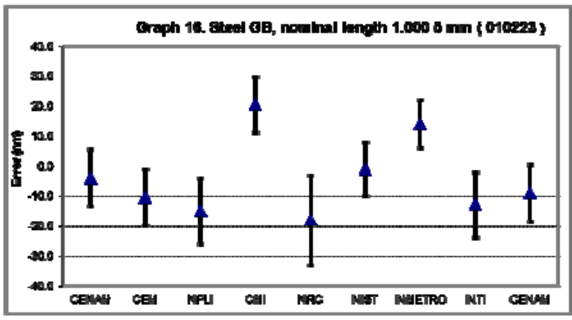
Nominal Value mm	Deviation from nominal length for Steel GB nm							
	CEM	NPLI	CMI	NRC	NIST	INMETRO	INTI	CENAM
1.000 5	-10.5	-15	20.5	-18	-1	14	-13	-4
5	16.5	18	60	25	41	48	13	35
7	-1.5	-12	37.5	-3	4	24	-14	-5
10	28	56	62.5	26	44	37	23	37
50	-29.5	22	29.5	11	13	-10	-14	7
75	-156	25	-39	-95	-100	-114	-119	-100
100	-103	-28	-8	-35	-41	-58	-49	-51

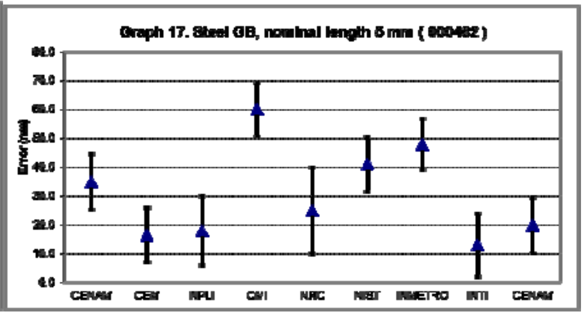
Table 8. Measurement results of the participants for the Steel GB.

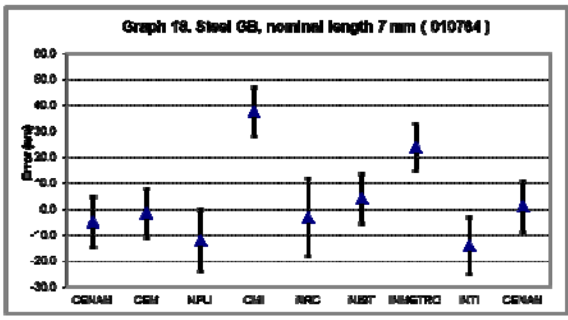
Nominal Value mm	Claimed standard uncertainties for Steel GB nm							
	CEM	NPLI	CMI	NRC	NIST	INMETRO	INTI	CENAM
1.000 5	9.4	11	9.4	15	9	8	11	9.6
5	9.5	12	9.4	15	9.4	9	11	9.6
7	9.5	12	9.4	15	9.5	9	11	9.7
10	9.6	13	9.5	15	9.8	9	11	9.8
50	11.9	16	10.9	25	13.5	16	16	14.8
75	14.4	23	12.5	16	15.7	22	21	19.4
100	17.3	26	14.5	16	18	28	27	24.5

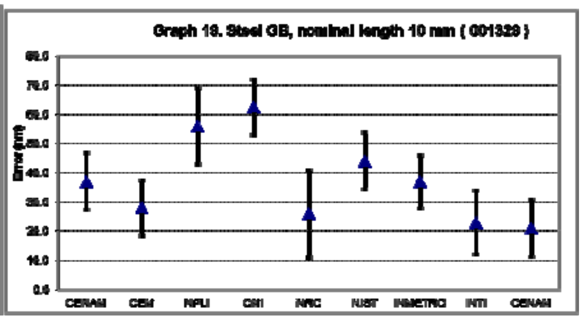
Table 9. Claimed standard uncertainties of the participants for Steel GB.

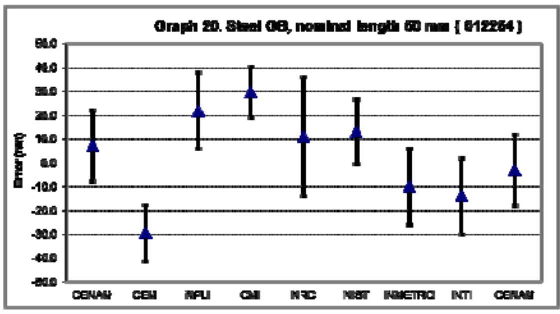


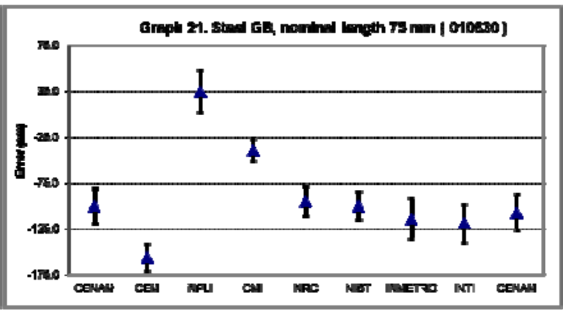


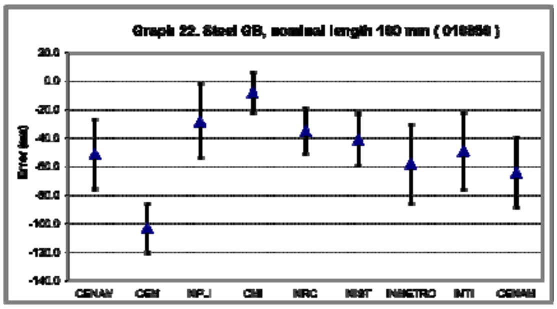












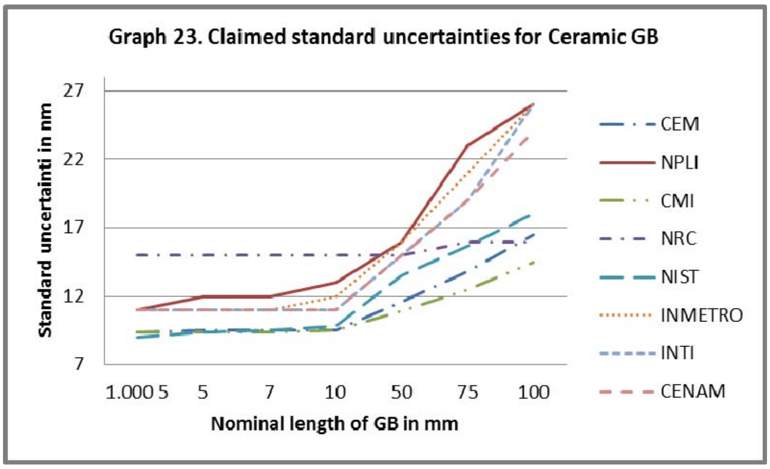
Tables 10 and **11** and **graphs 24** through **30**, show the deviations of the central length with respect to nominal values and their claimed standard measurement uncertainties of each participant for the seven ceramics GB; and **graph 23** show the claimed standard uncertainties of the participants.

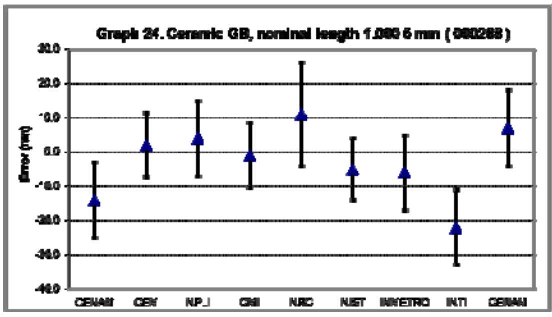
Nominal Value mm	Deviation from nominal length for Ceramics GB nm							
	CEM	NPLI	CMI	NRC	NIST	INMETRO	INTI	CENAM
1.000 5	2	4	-1	11	-5	-6	-22	-14
5	16	11	9	16	14	6	1	12
7	44	60	53.5	66	54	43	36	57
10	-19	-14	-18.5	-1	-11	-19	-19	-13
50	94	147	114.5	130	109	92	96	139
75	98.5	156	140.5	151	138	124	126	118
100	-50	28	2	2	-16	-45	-21	-34

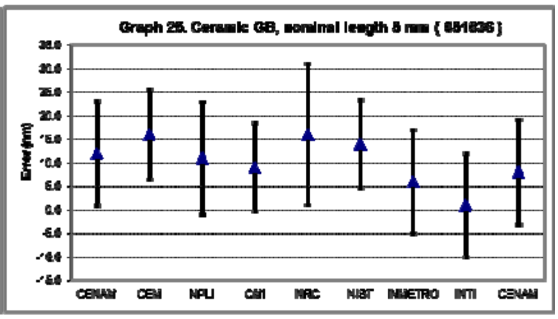
Table 10. Measurement results of the participants for Ceramics GB.

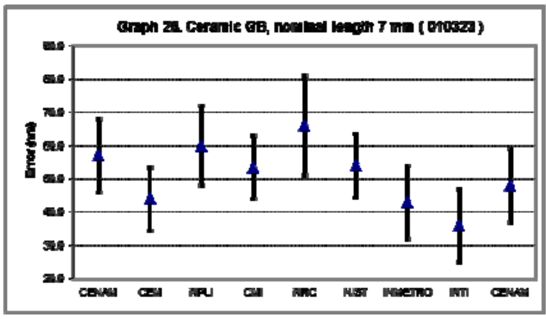
Nominal Value mm	Claimed standard uncertainties for Ceramics GB nm							
	CEM	NPLI	CMI	NRC	NIST	INMETRO	INTI	CENAM
1.000 5	9.4	11	9.4	15	9	11	11	11.1
5	9.5	12	9.4	15	9.4	11	11	11.1
7	9.5	12	9.4	15	9.5	11	11	11.1
10	9.5	13	9.5	15	9.8	12	11	11.2
50	11.6	16	10.9	15	13.5	16	15	15.3
75	13.8	23	12.5	16	15.7	21	19	19.3
100	16.5	26	14.5	16	18	26	26	23.8

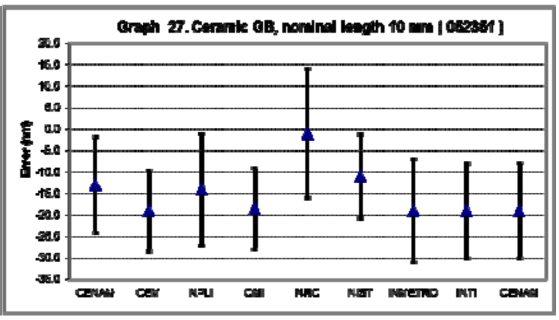
Table 11. Claimed standard uncertainties of the participants for Ceramics GB.

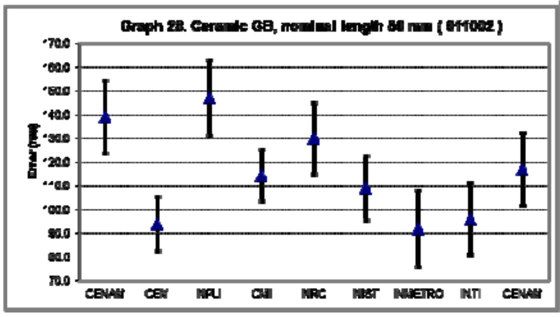

Graph 23. Claimed standard uncertainties for Ceramic GB
Standard uncertainti in nm
27
22
17
12
7
1.000 5
5
7
10
50
75
100
Nominal length of GB in mm
CEM
NPLI
CMI
NRC
NIST
INMETRO
INTI
CENAM

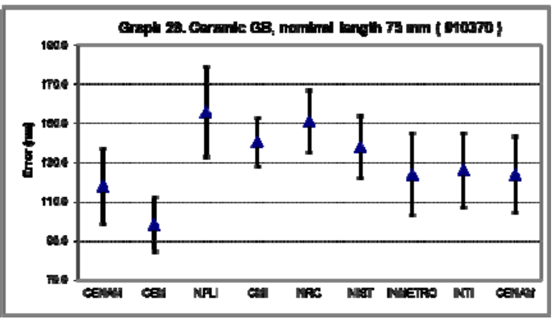

Graph 24. Ceramic GB, nominal length 1.000 5 mm
Error (nm)
CENAM
CEM
NPLI
CMI
NRC
NIST
INMETRO
INTI
CENAM

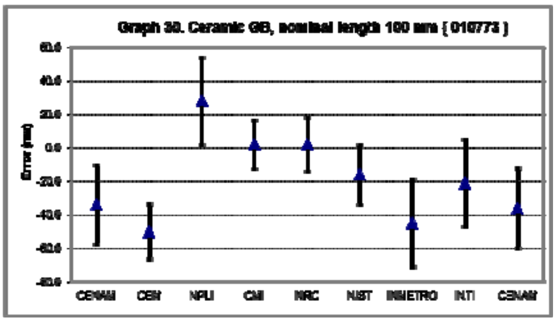

Graph 25. Ceramic GB, nominal length 5 mm
Error (nm)
CENAM
CEM
NPLI
CMI
NRC
NIST
INMETRO
INTI
CENAM


Graph 26. Ceramic GB, nominal length 7 mm
Error (nm)
CENAM
CEM
NPLI
CMI
NRC
NIST
INMETRO
INTI
CENAM


Graph 27. Ceramic GB, nominal length 10 mm
Error (nm)
CENAM
CEM
NPLI
CMI
NRC
NIST
INMETRO
INTI
CENAM

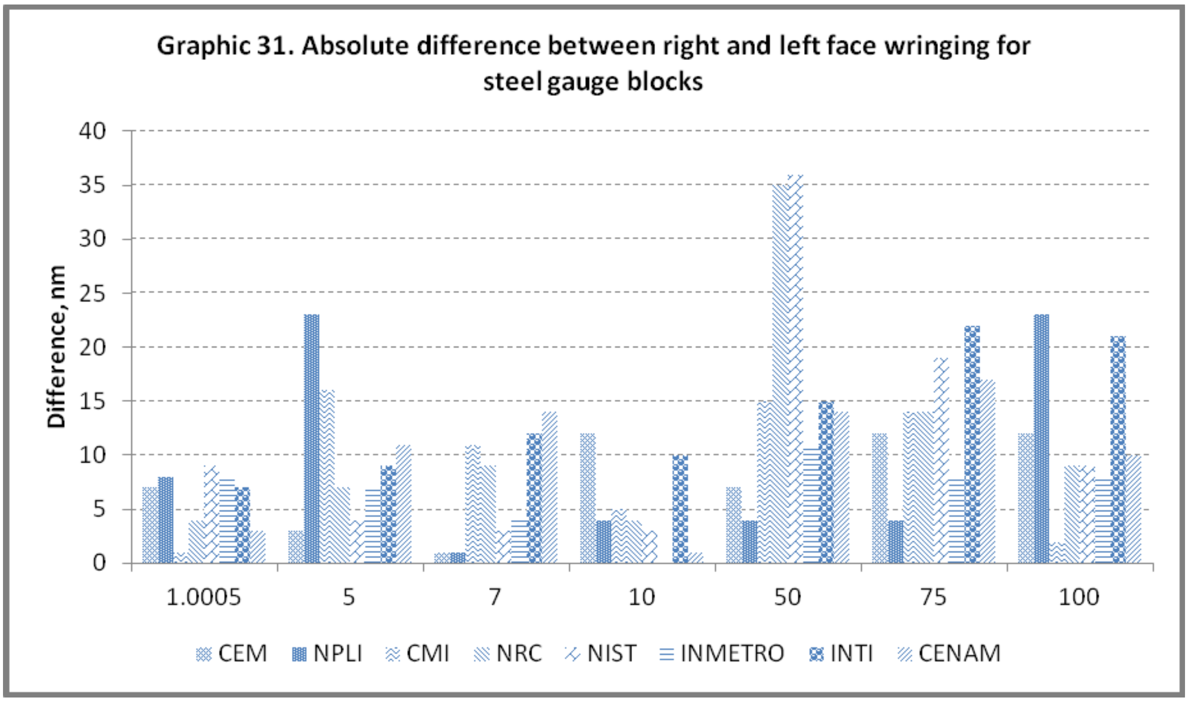

Graph 28. Ceramic GB, nominal length 50 mm
Error (nm)
CENAM
CEM
NPLI
CMI
NRC
NIST
INMETRO
INTI
CENAM

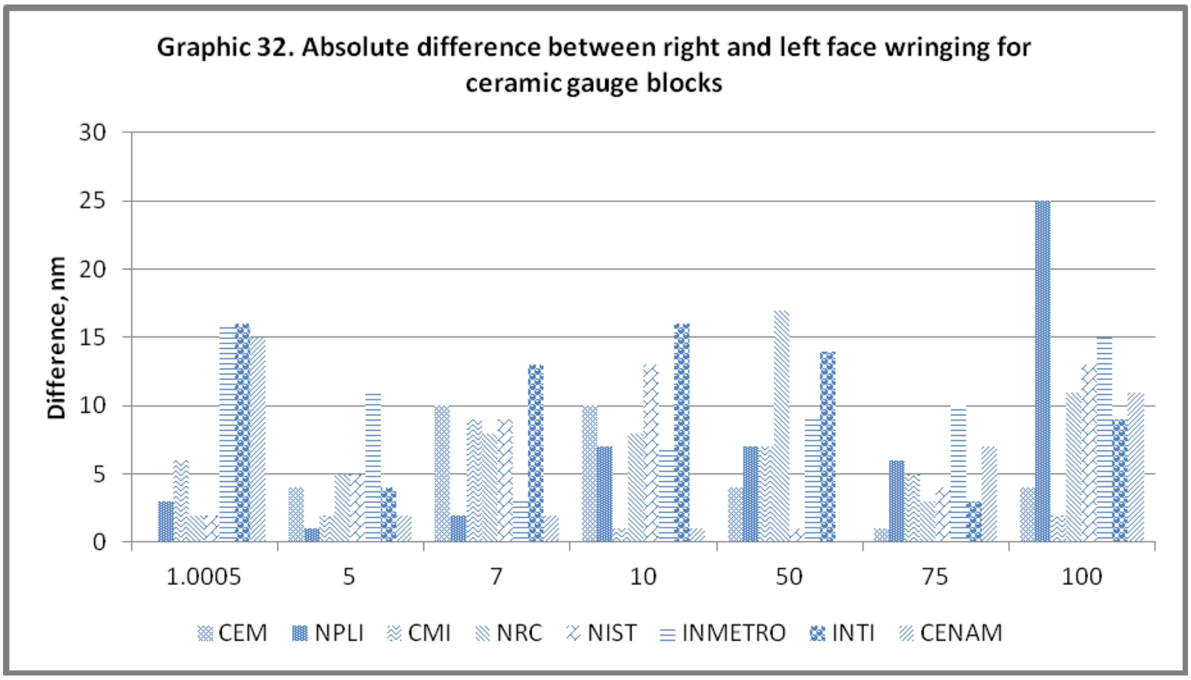

Graph 29. Ceramic GB, nominal length 75 mm
Error (nm)
CENAM
CEM
NPLI
CMI
NRC
NIST
INMETRO
INTI
CENAM


Graph 30. Ceramic GB, nominal length 100 mm
Error (nm)
CENAM
CEM
NPLI
CMI
NRC
NIST
INMETRO
INTI
CENAM

8.2 Measurement Difference of Length between the Two Measuring Faces

The protocol also asked to report the length measured on each of the measuring faces. Any difference is probably due to the quality of the wringing surfaces of the GB; the quality of the auxiliary wringing surface and the ability of the technician. Graphs 31 and 32 show the absolute values of these differences for the participants on every GB.





8.3 Phase-change Correction.

All participating laboratories applied the stack method to determine the phase-change correction for both materials except NIST. The latter laboratory assigns a value to each material and manufacturer of the GB according to a study they conducted in the 1990's using their reference platens [3]. **Table 12** summarizes the kind of platens the participants used and the phase-change correction values they submitted.

Participant	Steel Gauge Blocks		Ceramics Gauge Blocks	
	Platen material	Phase-change correction (nm)	Platen material	Phase-change correction (nm)
CEM	Steel B6(08/05A)	-38.1	Ceramics (TESA-111)	-4.9
NPLI	Steel (Id. 2)	-29	Steel (Id. 2)	-19
CMI[2]	Steel (Id. 4)	+12 (-12)	Steel (Id. 4)	-10
NRC	Fused Silica Id. Zygo	+50	Fused Silica Id. Zygo	+53
NIST	See [3]	+28.9	See [3]	+16.5
INMETRO	Quartz (17/19/18/14/15/18/11)	+51	Quartz (15/16/11/12/18)	+12
INTI	Steel (TESA-83)	-24	Steel (TESA-82)	-21
CENAM	Steel (TESA-86)	-19	Steel (TESA-86)	-16

Table 12. Phase-change correction of participants.

9. Analysis Method

9.1 Key Comparison Reference Value (KCRV) Determination

All usual parameters of the central tendency were calculated: the median, the simple mean and the inverse-variance weighted mean. All of these values appear on **Annex B**. However, the simple mean seemed the appropriate parameter to define the KCRV as all participants use the same calibration technique and state uncertainties that do not vary over a wide range.

9.2 Criteria to Determine the Largest Sub-set of "Consistent" Results to Compute the KCRV

The KCRV is determined, for each GB j, as the simple mean $\overline{e_j}$ of the largest subset of m_j participants which had "consistent" results[3]:

$$\overline{e_j} = \frac{1}{m_j}\sum_{i=1}^{m_j} e_{ij} \quad (1)$$

[2] CMI informed that they made a mistake on the phase change correction after they received DRAFT A. Their corrected value is indicated in parenthesis.

[3] Consistency is quoted because it is not checked in a rigorous statistical way but rather by the parameters that are described herein.

Where: e_{ij} is the deviation from nominal value of participant i, $i = 1,2,\ldots,m$ on GB j

m_j is the number of consistent results of the participants, $m_j \leq n$, where n is the total number of participants that measured GB j

The number of participants in this comparison is $n = 8$[4].

Two parameters are considered in the elimination process of inconsistent results on GB j:

- $|d_{ij}|$ The absolute deviation from the mean of participant i on GB j; and
- E_{Nij} The Normalized error of participant i on GB j defined as

$$E_{Nij} = \frac{|d_{ij}|}{U_{d_{ij}}} \quad (2)$$

Where $U(d_{ij})$ is the Expanded Uncertainty of deviation d_{ij}, computed as:

$$U(d_{ij}) = 2 \cdot \sqrt{u^2(e_{ij}) + u^2(\overline{e_j}) - \frac{2}{m_j} u^2(e_{ij})} \quad (3)$$

If laboratory i is taken in account in the calculation of the reference value; or

$$U(d_{ij}) = 2 \cdot \sqrt{u^2(e_{ij}) + u^2(\overline{e_j})} \quad (4)$$

If laboratory i has been eliminated from the calculation of the reference value.

If $E_{Nij} > 1$ it is considered that the result is not consistent.

To establish the largest subset of laboratories that had consistent results, an iterative elimination process of outliers was applied for each GB j. To start, all participants are considered into the calculation of the simple mean and the corresponding Normalized Errors are computed. Then the data are ordered according to their deviations d_{ij} from largest to smallest along with their corresponding E_{Nij} values. If the first participant on the list, with the largest deviation also has an $E_N > 1$, it is eliminated, $m = n - 1$ and the process is reiterated. The mean is recalculated and the remaining results are ordered according to their d_{ij} and along with their respective E_{Nij}. The process is repeated until no E_N values greater than 1 are found. It is important to note that the largest values of d_{ij} do not necessarily correspond to the largest values of E_N because the latter also depend on the declared uncertainty of the participant; although in most of cases they do. **Figure 5** shows the block diagram of the elimination process applied on each GB j.

9.3 KCRV Uncertainty

[4] CENAM, the pilot laboratory, contributes to the computation of the reference value only once with its first measurement.

The standard uncertainty corresponding to the reference value of GB *j* is given by the combined standard uncertainty of the simple mean, or internal uncertainty, of the m_j consistent results:

$$u_{int}\left(\bar{e}_j\right) = \frac{1}{m_j}\sqrt{\sum_{i=1}^{m_j} u^2\left(e_{ij}\right)} \qquad \text{where } m_j \le n \qquad (5)$$

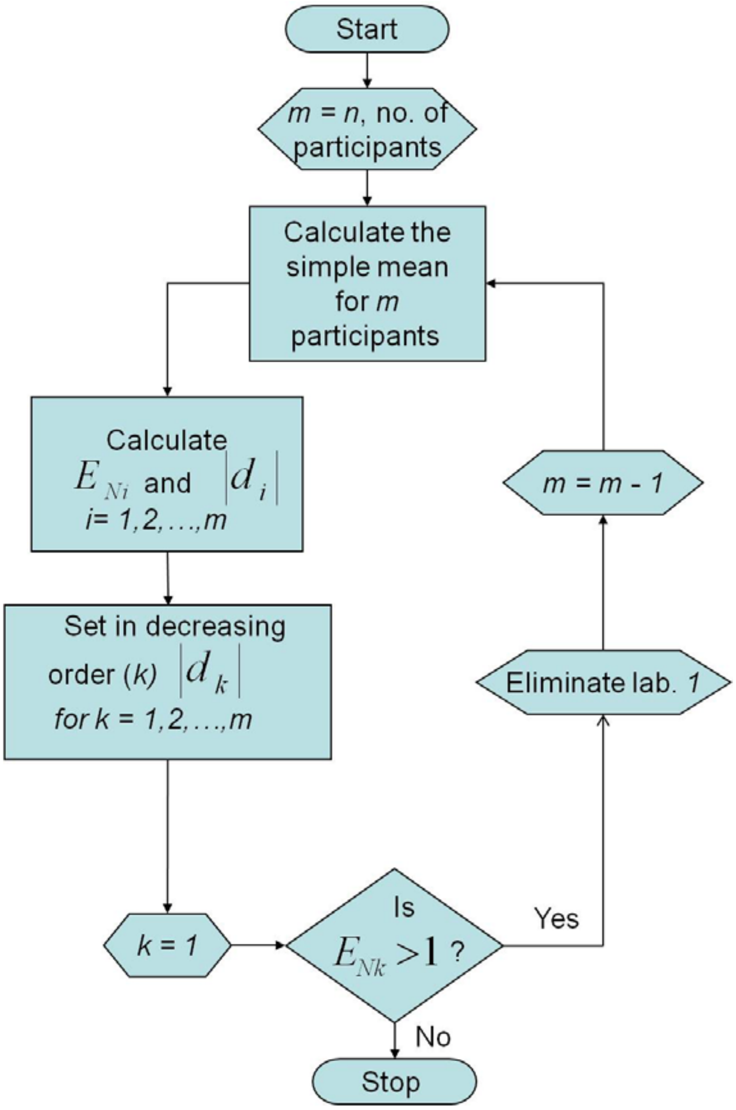


Figure 5. Flow chart showing the elimination process of inconsistent results for each GB *j*.

This value is used to calculate $u(\bar{e}_j)$ in equation (4) in the previous section: $u(\bar{e}_j) = u_{int}(\bar{e}_j)$.

1.1 Consistency of the Results taken into the KCRV Calculation

The standard deviation of the simple mean, $u(\bar{e}_j)$, or external uncertainty, of GB *j* is given by:

$$u_{ext}\left(\overline{e}_j\right) = \frac{1}{\sqrt{m_j}}\sqrt{\frac{\sum_{i=1}^{m_j}\left(e_{ij} - \overline{e}_j\right)^2}{m_j - 1}}$$ where $m_j \leq n$ **(6)**

If the measurement results and their uncertainties are consistent, the external uncertainty should be smaller or equal to the internal uncertainty. This is tested by means of the Birge Ratio defined as:

$$R_B = \frac{u_{ext}}{u_{int}}$$ where $R_B \leq 1$ for consistent results. **(7)**

10. Results of the Comparison

10.1 KCRV Determination

The Reference Values, $\overline{e}_j$, their Expanded Uncertainties, $U(\overline{e}_j)$, as well as the number of participants that contributed to the calculation, m_j, for the different GB j of both materials are shown in **Table 13**.

Reference Values						
Nominal	**Steel**			**Ceramics**		
Length	***Ref. Value,*** $\overline{e}_j$	$U(\overline{e}_j)$	m_j	***Ref. Value,*** $\overline{e}_j$	$U(\overline{e}_j)$	m_j
1.005	-10.3	9.0	6	-3.9	7.8	8
5	24.8	9.2	6	10.6	7.9	8
7	-5.3	9.2	6	51.7	7.9	8
10	35.9	8.5	7	-14.3	8.1	8
50	4.8	14.1	6	105.9	11.3	6
75	-105.6	17.0	5	136.2	13.9	7
100	-43.7	19.3	6	-23.1	15.6	7

Table 13. Reference values (simple mean of largest sub-set of "consistent" results) deviations from Nominal Value with Expanded Uncertainty and number of values contributing to the calculation of the Reference Value Computation (m_j) for both steel and ceramics GB.

The elimination process of outliers is shown for each GB j in **Annex A**.

10.2 Participants Results.

Tables 14 and **15** show the differences of the results of the participants with respect to the Reference Values of each GB j, d_{ij} ; along with the Expanded Uncertainty of these differences, $U(d_{ij})$; and the corresponding Normalized Error, E_{Nij}.

NMI ($i\rightarrow$)	**CEM**			**CENAM**			**CMI**			**INMETRO**		
Nom L ($j\downarrow$)	d_{ij}	$U(d_{ij})$	E_{Nij}	d_{ij}	$U(d_{ij})$	E_{Nij}	d_{ij}	$U(d_{ij})$	E_{Nij}	d_{ij}	$U(d_{ij})$	E_{Nij}
1.000 5	-0.3	18.8	0.0	6.3	19.2	0.3	30.8	18.8	**1.5**	24.3	16.0	**1.3**
5	-8.3	19.0	0.5	10.3	19.2	0.6	35.3	18.8	**1.7**	23.3	18.0	**1.2**
7	3.8	19.0	0.2	0.3	19.4	0.0	42.8	18.8	**2.0**	29.3	18.0	**1.4**
10	-7.9	19.2	0.4	1.1	19.6	0.1	26.6	19.0	**1.3**	1.1	18.0	0.1
50	-34.3	23.8	**1.2**	2.2	29.6	0.1	24.7	21.8	0.9	-14.8	32.0	0.5
75	-50.4	28.8	**1.6**	5.6	38.8	0.2	66.6	25.0	**2.3**	-8.4	44.0	0.2
100	-59.3	34.6	**1.5**	-7.3	49.0	0.2	35.7	29.0	**1.0**	-14.3	56.0	0.3

Table 14 A. Deviation from reference value for each GB, d_{ij}; claimed expanded uncertainty ($k=2$), U_{ij}; and Normalized Error E_{Nij} of the **Steel GB** for the first four participants.

NMI (i→)	INTI			NIST			NPLI			NRC		
Nom L (j↓)	d_{ij}	$U(d_{ij})$	E_{Nij}	d_{ij}	$U(d_{ij})$	E_{Nij}	d_{ij}	$U(d_{ij})$	E_{Nij}	d_{ij}	$U(d_{ij})$	E_{Nij}
1.000 5	-2.8	22.0	0.1	9.3	18.0	0.5	-4.8	22.0	0.2	-7.8	30.0	0.3
5	-11.8	22.0	0.6	16.3	18.8	0.9	-6.8	24.0	0.3	0.3	30.0	0.0
7	-8.8	22.0	0.4	9.3	19.0	0.5	-6.8	24.0	0.3	2.3	30.0	0.1
10	-12.9	22.0	0.6	8.1	19.6	0.4	20.1	26.0	0.9	-9.9	30.0	0.4
50	-18.8	32.0	0.6	8.2	27.0	0.3	17.2	32.0	0.6	6.2	50.0	0.1
75	-13.4	42.0	0.4	5.6	31.4	0.2	130.6	46.0	**2.7**	10.6	32.0	0.4
100	-5.3	54.0	0.1	2.7	36.0	0.1	15.7	52.0	0.3	8.7	32.0	0.3

Table 14 B. Deviation from reference value for each GB, d_{ij}; claimed standard uncertainty, U_{ij}; and Normalized Error E_{Nij} of the **Steel GB** for the last four participants**.**

NMI (i→)	CEM			CENAM			CMI			INMETRO		
Nom L (j↓)	d_{ij}	$U(d_{ij})$	E_{Nij}	d_{ij}	$U(d_{ij})$	E_{Nij}	d_{ij}	$U(d_{ij})$	E_{Nij}	d_{ij}	$U(d_{ij})$	E_{Nij}
1.000 5	5.9	18.8	0.3	-10.1	22.2	0.5	2.9	18.8	0.2	-2.1	22.0	0.1
5	5.4	19.0	0.3	1.4	22.2	0.1	-1.6	18.8	0.1	-4.6	22.0	0.2
7	-7.7	19.0	0.4	5.3	22.2	0.3	1.8	18.8	0.1	-8.7	22.0	0.4
10	-4.7	19.0	0.3	1.3	22.4	0.1	-4.2	19.0	0.2	-4.7	24.0	0.2
50	-11.9	23.2	0.5	33.1	30.6	**1.1**	8.6	21.8	0.4	-13.9	32.0	0.5
75	-37.7	27.6	**1.2**	-18.2	38.6	0.5	4.3	25.0	0.2	-12.2	42.0	0.3
100	-26.9	33	0.8	-10.9	47.6	0.3	25.1	29.0	0.9	-21.9	52.0	0.5

Table 15 A. Deviation from reference value for each GB, d_{ij}; claimed standard uncertainty, U_{ij}; and Normalized Error E_{Nij} of the **Ceramics GB** for the first four participants

NMI (i→)	INTI			NIST			NPLI			NRC		
Nom L (j↓)	d_{ij}	$U(d_{ij})$	E_{Nij}	d_{ij}	$U(d_{ij})$	E_{Nij}	d_{ij}	$U(d_{ij})$	E_{Nij}	d_{ij}	$U(d_{ij})$	E_{Nij}
1.000 5	-18.1	22.0	0.9	-1.1	18.0	0.1	7.9	22.0	0.4	14.9	30.0	0.5
5	-9.6	22.0	0.5	3.4	18.8	0.2	0.4	24.0	0.0	5.4	30.0	0.2
7	-15.7	22.0	0.8	2.3	19.0	0.1	8.3	24.0	0.4	14.3	30.0	0.5
10	-4.7	22.0	0.2	3.3	19.6	0.2	0.3	26.0	0.0	13.3	30.0	0.5
50	-9.9	30.0	0.4	3.1	27.0	0.1	41.1	32.0	**1.3**	24.1	30.0	0.9
75	-10.2	38.0	0.3	1.8	31.4	0.1	19.8	46.0	0.5	14.8	32.0	0.5
100	2.1	52.0	0.0	7.1	36.0	0.2	51.1	52.0	0.9	25.1	32.0	0.8

Table 15 B. Deviation from reference value for each GB, d_{ij}; claimed standard uncertainty, U_{ij}; and Normalized Error E_{Nij} of the **Ceramics GB** for the last four participants.

The bilateral equivalences between every two laboratories were also calculated. For this purpose the following equations were applied:

$$d_{kl} = d_l - d_k \quad \text{where } k = 1, 2,\ldots,n \text{ ; } l = 1, 2,\ldots,n \text{ ; and } k \neq l \qquad (8)$$

$$U_{kl} = 2 \cdot \sqrt{u_k^2 + u_l^2} \qquad (9)$$

$$E_{kl} = |d_{kl}| / U_{kl} \qquad (10)$$

Tables 16 through **22**, show the bilateral equivalences between laboratories for the **Steel GB**; and **tables 23** through **29** show the bilateral equivalences for the **Ceramics GB**.

11. Discussion and Conclusions

11.1 Discussion

- The organization of the comparison started in January 2007 and the artifacts started circulation on November of the same year. There were quite a few delays on the original schedule mainly due to problems on customs clearance of the artifacts. Therefore, the circulation ended until mid-April, 2010; a time span of almost two and a half years for a total number of nine participants including the pilots measurements at the beginning and at the end.
- The South African laboratory, NMISA, had problems and did not send-in measurement results. They requested a new opportunity to measure the GB again. However, as these GB were also going to be used at a subsequent exercise of Mechanical Comparison, it was not possible unfortunately. In **Annex C 1** the correspondence with this laboratory is shown.
- Draft A was sent-out for review of the participants on May 2010. Only František Dvořáček from CMI sent in a letter reckoning they made a mistake on the phase change correction sign and asked to correct it. The correspondence appears in **Annex C 2. Annex D** shows the comparison Analysis considering this correction. It is evident that there was a sign mistake and considering this correction CMI results are consistent six out of seven.
- The Draft B version of the report could not be released until present, because the same artifacts were used for a second comparison of GB measured by mechanical comparison and the measured values could not be disclosed until all participants of the second comparison had sent-in their results. This second comparison had 15 participants and CENAM performed the final measurements on May 2011.

11.2 Conclusions

- From Section 7 we observe that there were no appreciable changes on the measurements performed by the pilot laboratory of the ensemble of the GB of both materials over the last five years. Even though some drift may be appreciated on the steel GB during their first year of their history, the values shown prove they reached stability since 2005 approximately. Therefore, it can be assumed that the artifacts behaved adequately during the comparison exercise and that the exercise was valid.

- Alternate statistical estimators and parameters were computed and are shown on **Annex B.** Although the analysis chosen used the simple mean to compute the KCRV. The results are shown for completeness and lead to the same conclusions.
- The elimination process described in section 9 proved to be adequate as the Birge ratio becomes lower than 1 once the outliers are eliminated, therefore proving consistency of results. The detailed elimination process for each GB is shown in **Annex A**.
- In general, the scatter of results was smaller for the Ceramics GB than for the Steel GB. This can partially be explained because of the Phase-Change Correction error of CMI, but even taking in account this correction (see **Annex D**) the results for the Steel GB are still more disperse. This may be due to the fact that Steel GB are more prone to scratches, rust or damage.
- The results of most laboratories were satisfactory in most artifacts, but we would like to make a few comments on those results that had $E_n > 1$.
- The case of CMI for the Steel GB has already been discussed.
- For the longer GB of steel and one of Ceramic, CEM had $E_n > 1$ and their results are always under the mean which might suggest the investigation systematic effects that may grow with length.
- INMETRO had En > 1 for some of the shorter steel GB (i.e. for nominal lengths of 1.0005, 5.0 and 7.0 mm). INMETRO believes that the positive deviations from the mean may have been caused by wringing problems associated with scratches on the surfaces.
- NPLI had some problems with the 75 mm steel GB and the 50 mm ceramics. It is difficult to suggest a possible reason, but a frequent one is temperature. An investigation on the cause should also be carried out.
- In general the results of all participants were satisfactory which prove their technical competency.

12. Acknowledgements

We would like to acknowledge:

- SIM WG.4 Length and SIM Technical Committee for having funded the purchase of the fourteen GB to carry this comparison.
- In an anonymous way, the technicians and colleagues from our different institutions that contributed directly or indirectly to the measurements of the artifacts in this comparison.
- And specifically from CENAM, the pilot laboratory, Juan Carlos Zárraga who repeated the measurements made by Carlos Colín.
- Our colleague from the Dimensional Metrology Division (DMD), Armando López Celis who gave us a hand with the spread sheet analysis of the results.

NMI (*k*→)	CEM			CENAM			CMI			INMETRO			INTI			NIST			NPLI		
NMI (*l*↓)	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}
CEM																					
CENAM	6.5	26.9	0.2																		
CMI	31.0	26.6	**1.2**	24.5	26.9	0.9															
INMETRO	24.5	24.7	1.0	18.0	25.0	0.7	-6.5	24.7	0.3												
INTI	-2.5	28.9	0.1	-9.0	29.2	0.3	-33.5	28.9	**1.2**	-27.0	27.2	1.0									
NIST	9.5	26.0	0.4	3.0	26.3	0.1	-21.5	26.0	0.8	-15.0	24.1	0.6	12.0	28.4	0.4						
NPLI	-4.5	28.9	0.2	-11.0	29.2	0.4	-35.5	28.9	**1.2**	-29.0	27.2	**1.1**	-2.0	31.1	0.1	-14.0	28.4	0.5			
NRC	-7.5	35.4	0.2	-14.0	35.6	0.4	-38.5	35.4	**1.1**	-32.0	34.0	0.9	-5.0	37.2	0.1	-17.0	35.0	0.5	-3.0	37.2	0.1

Table 16. Bilateral equivalences for the 1.0005 mm Steel GB.

NMI (*k*→)	CEM			CENAM			CMI			INMETRO			INTI			NIST			NPLI		
NMI (*l*↓)	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}
CEM																					
CENAM	18.5	27.0	0.7																		
CMI	43.5	26.7	**1.6**	25.0	26.9	0.9															
INMETRO	31.5	26.2	**1.2**	13.0	26.3	0.5	-12.0	26.0	0.5												
INTI	-3.5	29.1	0.1	-22.0	29.2	0.8	-47.0	28.9	**1.6**	-35.0	28.4	**1.2**									
NIST	24.5	26.7	0.9	6.0	26.9	0.2	-19.0	26.6	0.7	-7.0	26.0	0.3	28.0	28.9	1.0						
NPLI	1.5	30.6	0.0	-17.0	30.7	0.6	-42.0	30.5	**1.4**	-30.0	30.0	1.0	5.0	32.6	0.2	-23.0	30.5	0.8			
NRC	8.5	35.5	0.2	-10.0	35.6	0.3	-35.0	35.4	1.0	-23.0	35.0	0.7	12.0	37.2	0.3	-16.0	35.4	0.5	7.0	38.4	0.2

Table 17 Bilateral equivalences for the 5 mm Steel GB.

NMI (*k*→)	**CEM**			**CENAM**			**CMI**			**INMETRO**			**INTI**			**NIST**			**NPLI**		
NMI (*l*↓)	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}
CEM																					
CENAM	-3.5	27.2	0.1																		
CMI	39.0	26.7	**1.5**	42.5	27.0	**1.6**															
INMETRO	25.5	26.2	1.0	29.0	26.5	**1.1**	-13.5	26.0	0.5												
INTI	-12.5	29.1	0.4	-9.0	29.3	0.3	-51.5	28.9	**1.8**	-38.0	28.4	**1.3**									
NIST	5.5	26.9	0.2	9.0	27.2	0.3	-33.5	26.7	**1.3**	-20.0	26.2	0.8	18.0	29.1	0.6						
NPLI	-10.5	30.6	0.3	-7.0	30.9	0.2	-49.5	30.5	**1.6**	-36.0	30.0	**1.2**	2.0	32.6	0.1	-16.0	30.6	0.5			
NRC	-1.5	35.5	0.0	2.0	35.7	0.1	-40.5	35.4	**1.1**	-27.0	35.0	0.8	11.0	37.2	0.3	-7.0	35.5	0.2	9.0	38.4	0.2

Table 18. Bilateral equivalences for the 7 mm Steel GB.

NMI (*k*→)	**CEM**			**CENAM**			**CMI**			**INMETRO**			**INTI**			**NIST**			**NPLI**		
NMI (*l*↓)	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}
CEM																					
CENAM	9.0	27.4	0.3																		
CMI	34.5	27.0	**1.3**	25.5	27.3	0.9															
INMETRO	9.0	26.3	0.3	0.0	26.6	0.0	-25.5	26.2	1.0												
INTI	-5.0	29.2	0.2	-14.0	29.5	0.5	-39.5	29.1	**1.4**	-14.0	28.4	0.5									
NIST	16.0	27.4	0.6	7.0	27.7	0.3	-18.5	27.3	0.7	7.0	26.6	0.3	21.0	29.5	0.7						
NPLI	28.0	32.3	0.9	19.0	32.6	0.6	-6.5	32.2	0.2	19.0	31.6	0.6	33.0	34.1	1.0	12.0	32.6	0.4			
NRC	-2.0	35.6	0.1	-11.0	35.8	0.3	-36.5	35.5	**1.0**	-11.0	35.0	0.3	3.0	37.2	0.1	-18.0	35.8	0.5	-30.0	39.7	0.8

Table 19. Bilateral equivalences for the 10 mm Steel GB.

NMI (*k*→)	**CEM**			**CENAM**			**CMI**			**INMETRO**			**INTI**			**NIST**			**NPLI**		
NMI (*l*↓)	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}
CEM																					
CENAM	36.5	38.0	1.0																		
CMI	59.0	32.3	**1.8**	22.5	36.8	0.6															
INMETRO	19.5	39.9	0.5	-17.0	43.6	0.4	-39.5	38.7	**1.0**												
INTI	15.5	39.9	0.4	-21.0	43.6	0.5	-43.5	38.7	**1.1**	-4.0	45.3	0.1									
NIST	42.5	36.0	**1.2**	6.0	40.1	0.1	-16.5	34.7	0.5	23.0	41.9	0.5	27.0	41.9	0.6						
NPLI	51.5	39.9	**1.3**	15.0	43.6	0.3	-7.5	38.7	0.2	32.0	45.3	0.7	36.0	45.3	0.8	9.0	41.9	0.2			
NRC	40.5	55.4	0.7	4.0	58.1	0.1	-18.5	54.5	0.3	21.0	59.4	0.4	25.0	59.4	0.4	-2.0	56.8	0.0	-11.0	59.4	0.2

Table 20. Bilateral equivalences for the 50 mm Steel GB.

NMI (*k*→)	**CEM**			**CENAM**			**CMI**			**INMETRO**			**INTI**			**NIST**			**NPLI**		
NMI (*l*↓)	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}
CEM																					
CENAM	56.0	48.3	**1.2**																		
CMI	117.0	38.1	**3.1**	61.0	46.2	**1.3**															
INMETRO	42.0	52.6	0.8	-14.0	58.7	0.2	-75.0	50.6	**1.5**												
INTI	37.0	50.9	0.7	-19.0	57.2	0.3	-80.0	48.9	**1.6**	-5.0	60.8	0.1									
NIST	56.0	42.6	**1.3**	0.0	49.9	0.0	-61.0	40.1	**1.5**	14.0	54.1	0.3	19.0	52.4	0.4						
NPLI	181.0	54.3	**3.3**	125.0	60.2	**2.1**	64.0	52.4	**1.2**	139.0	63.7	**2.2**	144.0	62.3	**2.3**	125.0	55.7	**2.2**			
NRC	61.0	43.1	**1.4**	5.0	50.3	0.1	-56.0	40.6	**1.4**	19.0	54.4	0.3	24.0	52.8	0.5	5.0	44.8	0.1	-120.0	56.0	**2.1**

Table 21 Bilateral equivalences for the 75 mm steel GB.

NMI (*k*→)	**CEM**			**CENAM**			**CMI**			**INMETRO**			**INTI**			**NIST**			**NPLI**		**NRC**
NMI (*l*↓)	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}
CEM																					
CENAM	52.0	60.0	0.9																		
CMI	95.0	45.1	**2.1**	43.0	56.9	0.8															
INMETRO	45.0	65.8	0.7	-7.0	74.4	0.1	-50.0	63.1	0.8												
INTI	54.0	64.1	0.8	2.0	72.9	0.0	-41.0	61.3	0.7	9.0	77.8	0.1									
NIST	62.0	49.9	**1.2**	10.0	60.8	0.2	-33.0	46.2	0.7	17.0	66.6	0.3	8.0	64.9	0.1						
NPLI	75.0	62.5	**1.2**	23.0	71.4	0.3	-20.0	59.5	0.3	30.0	76.4	0.4	21.0	75.0	0.3	13.0	63.2	0.2			
NRC	68.0	47.1	**1.4**	16.0	58.5	0.3	-27.0	43.2	0.6	23.0	64.5	0.4	14.0	62.8	0.2	6.0	48.2	0.1	-7.0	61.1	0.1

Table 22. Bilateral equivalences for the 100 mm steel GB.

NMI (*k*→)	**CEM**			**CENAM**			**CMI**			**INMETRO**			**INTI**			**NIST**			**NPLI**		
NMI (*l*↓)	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}
CEM																					
CENAM	-16.0	29.1	0.6																		
CMI	-3.0	26.6	0.1	13.0	29.1	0.4															
INMETRO	-8.0	28.9	0.3	8.0	31.3	0.3	-5.0	28.9	0.2												
INTI	-24.0	28.9	0.8	-8.0	31.3	0.3	-21.0	28.9	0.7	-16.0	31.1	0.5									
NIST	-7.0	26.0	0.3	9.0	28.6	0.3	-4.0	26.0	0.2	1.0	28.4	0.0	17.0	28.4	0.6						
NPLI	2.0	28.9	0.1	18.0	31.3	0.6	5.0	28.9	0.2	10.0	31.1	0.3	26.0	31.1	0.8	9.0	28.4	0.3			
NRC	9.0	35.4	0.3	25.0	37.3	0.7	12.0	35.4	0.3	17.0	37.2	0.5	33.0	37.2	0.9	16.0	35.0	0.5	7.0	37.2	0.2

Table 23. Bilateral equivalences for the 1.0005 mm Ceramics GB.

NMI ($k\rightarrow$)	CEM			CENAM			CMI			INMETRO			INTI			NIST			NPLI		
NMI ($l\downarrow$)	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}
CEM																					
CENAM	-4.0	29.2	0.1																		
CMI	-7.0	26.7	0.3	-3.0	29.1	0.1															
INMETRO	-10.0	29.1	0.3	-6.0	31.3	0.2	-3.0	28.9	0.1												
INTI	-15.0	29.1	0.5	-11.0	31.3	0.4	-8.0	28.9	0.3	-5.0	31.1	0.2									
NIST	-2.0	26.7	0.1	2.0	29.1	0.1	5.0	26.6	0.2	8.0	28.9	0.3	13.0	28.9	0.4						
NPLI	-5.0	30.6	0.2	-1.0	32.7	0.0	2.0	30.5	0.1	5.0	32.6	0.2	10.0	32.6	0.3	-3.0	30.5	0.1			
NRC	0.0	35.5	0.0	4.0	37.3	0.1	7.0	35.4	0.2	10.0	37.2	0.3	15.0	37.2	0.4	2.0	35.4	0.1	5.0	38.4	0.1

Table 24. Bilateral equivalences for the 5 mm Ceramics GB.

NMI ($k\rightarrow$)	CEM			CENAM			CMI			INMETRO			INTI			NIST			NPLI		
NMI ($l\downarrow$)	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}
CEM																					
CENAM	13.0	29.2	0.4																		
CMI	9.5	26.7	0.4	-3.5	29.1	0.1															
INMETRO	-1.0	29.1	0.0	-14.0	31.3	0.4	-10.5	28.9	0.4												
INTI	-8.0	29.1	0.3	-21.0	31.3	0.7	-17.5	28.9	0.6	-7.0	31.1	0.2									
NIST	10.0	26.9	0.4	-3.0	29.2	0.1	0.5	26.7	0.0	11.0	29.1	0.4	18.0	29.1	0.6						
NPLI	16.0	30.6	0.5	3.0	32.7	0.1	6.5	30.5	0.2	17.0	32.6	0.5	24.0	32.6	0.7	6.0	30.6	0.2			
NRC	22.0	35.5	0.6	9.0	37.3	0.2	12.5	35.4	0.4	23.0	37.2	0.6	30.0	37.2	0.8	12.0	35.5	0.3	6.0	38.4	0.2

Table 25. Bilateral equivalences for the 7 mm Ceramics GB.

NMI (*k*→)	**CEM**			**CENAM**			**CMI**			**INMETRO**			**INTI**			**NIST**			**NPLI**		
NMI (*l*↓)	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}
CEM																					
CENAM	6.0	29.4	0.2																		
CMI	0.5	26.9	0.0	-5.5	29.4	0.2															
INMETRO	0.0	30.6	0.0	-6.0	32.8	0.2	-0.5	30.6	0.0												
INTI	0.0	29.1	0.0	-6.0	31.4	0.2	-0.5	29.1	0.0	0.0	32.6	0.0									
NIST	8.0	27.3	0.3	2.0	29.8	0.1	7.5	27.3	0.3	8.0	31.0	0.3	8.0	29.5	0.3						
NPLI	5.0	32.2	0.2	-1.0	34.3	0.0	4.5	32.2	0.1	5.0	35.4	0.1	5.0	34.1	0.1	-3.0	32.6	0.1			
NRC	18.0	35.5	0.5	12.0	37.4	0.3	17.5	35.5	0.5	18.0	38.4	0.5	18.0	37.2	0.5	10.0	35.8	0.3	13.0	39.7	0.3

Table 26. Bilateral equivalences for the 10 mm Ceramics GB

NMI (*k*→)	**CEM**			**CENAM**			**CMI**			**INMETRO**			**INTI**			**NIST**			**NPLI**		
NMI (*l*↓)	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}
CEM																					
CENAM	45.0	38.4	**1.2**																		
CMI	20.5	31.8	0.6	-24.5	37.6	0.7															
INMETRO	-2.0	39.5	0.1	-47.0	44.3	**1.1**	-22.5	38.7	0.6												
INTI	2.0	37.9	0.1	-43.0	42.9	**1.0**	-18.5	37.1	0.5	4.0	43.9	0.1									
NIST	15.0	35.6	0.4	-30.0	40.8	0.7	-5.5	34.7	0.2	17.0	41.9	0.4	13.0	40.4	0.3						
NPLI	53.0	39.5	**1.3**	8.0	44.3	0.2	32.5	38.7	0.8	55.0	45.3	**1.2**	51.0	43.9	**1.2**	38.0	41.9	0.9			
NRC	36.0	37.9	0.9	-9.0	42.9	0.2	15.5	37.1	0.4	38.0	43.9	0.9	34.0	42.4	0.8	21.0	40.4	0.5	-17.0	43.9	0.4

Table 27. Bilateral equivalences for the 50 mm Ceramics GB.

NMI (*k*→)	CEM			CENAM			CMI			INMETRO			INTI			NIST			NPLI		
NMI (*l*↓)	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}
CEM																					
CENAM	19.5	47.5	0.4																		
CMI	42.0	37.2	**1.1**	22.5	46.0	0.5															
INMETRO	25.5	50.3	0.5	6.0	57.0	0.1	-16.5	48.9	0.3												
INTI	27.5	47.0	0.6	8.0	54.2	0.1	-14.5	45.5	0.3	2.0	56.6	0.0									
NIST	39.5	41.8	0.9	20.0	49.8	0.4	-2.5	40.1	0.1	14.0	52.4	0.3	12.0	49.3	0.2						
NPLI	57.5	53.6	**1.1**	38.0	60.0	0.6	15.5	52.4	0.3	32.0	62.3	0.5	30.0	59.7	0.5	18.0	55.7	0.3			
NRC	52.5	42.3	**1.2**	33.0	50.1	0.7	10.5	40.6	0.3	27.0	52.8	0.5	25.0	49.7	0.5	13.0	44.8	0.3	-5.0	56.0	0.1

Table 28. Bilateral equivalences for the 75 mm Ceramics GB.

NMI (*k*→)	CEM			CENAM			CMI			INMETRO			INTI			NIST			NPLI		
NMI (*l*↓)	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}	d_{kl}	$U(d_{kl})$	E_{Nkl}
CEM																					
CENAM	16.0	57.9	0.3																		
CMI	52.0	43.9	**1.2**	36.0	55.7	0.6															
INMETRO	5.0	61.6	0.1	-11.0	70.5	0.2	-47.0	59.5	0.8												
INTI	29.0	61.6	0.5	13.0	70.5	0.2	-23.0	59.5	0.4	24.0	73.5	0.3									
NIST	34.0	48.8	0.7	18.0	59.7	0.3	-18.0	46.2	0.4	29.0	63.2	0.5	5.0	63.2	0.1						
NPLI	78.0	61.6	**1.3**	62.0	70.5	0.9	26.0	59.5	0.4	73.0	73.5	1.0	49.0	73.5	0.7	44.0	63.2	0.7			
NRC	52.0	46.0	**1.1**	36.0	57.4	0.6	0.0	43.2	0.0	47.0	61.1	0.8	23.0	61.1	0.4	18.0	48.2	0.4	-26.0	61.1	0.4

Table 29. Bilateral equivalences for the 100 mm Ceramics GB.

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Annex A Elimination of Inconsistent Results

1.000 5 mm, Steel GB								
Nom. length	*n = 8*	*Elim. CMI*	Nom. length	*n = 7*	*Elim. INMETRO*	Nom. length	*n = 6*	*STOP*
NMI	d_i	E_N	NMI	d_i	E_N	NMI	d_i	E_n
CMI	23.9	**1.3**	INMETRO	20.8	**1.3**	NIST	9.3	0.5
INMETRO	17.4	**1.1**	NRC	11.2	0.4	NRC	7.8	0.3
NRC	14.6	0.5	NPLI	8.2	0.4	CENAM	6.3	0.3
NPLI	11.6	0.6	INTI	6.2	0.3	NPLI	4.8	0.2
INTI	9.6	0.5	NIST	5.8	0.3	INTI	2.8	0.1
CEM	7.1	0.4	CEM	3.7	0.2	CEM	0.3	0.0
NIST	2.4	0.1	CENAM	2.8	0.2	**CMI**	**30.8**	**1.5**
CENAM	0.6	0.0				**INMETRO**	**24.3**	**1.3**

Table A1. Consecutive elimination of two inconsistent results to arrive to a set of 6 consistent ones for the 1.0005 mm Steel GB.

5 mm, Steel GB								
Nom. length	*n = 8*	*Elim. CMI*	Nom. length	*n = 7*	*Elim. INMETRO*	Nom. length	*n = 6*	*STOP*
NMI	d_i	E_N	NMI	d_i	E_N	NMI	d_i	E_N
CMI	27.9	**1.6**	INMETRO	19.9	**1.2**	NIST	16.3	0.9
INTI	19.1	0.9	INTI	15.1	0.7	INTI	11.8	0.6
INMETRO	15.9	0.9	NIST	12.9	0.7	CENAM	10.3	0.6
CEM	15.6	0.9	CEM	11.6	0.6	CEM	8.3	0.5
NPLI	14.1	0.6	NPLI	10.1	0.5	NPLI	6.8	0.3
NIST	8.9	0.5	CENAM	6.9	0.4	NRC	0.3	0.0
NRC	7.1	0.3	NRC	3.1	0.1	**CMI**	**35.3**	**1.7**
CENAM	2.9	0.2				**INMETRO**	**23.3**	**1.2**

Table A2. Consecutive elimination of two inconsistent results to arrive to a set of 6 consistent ones for the 5 mm Steel GB.

7 mm, Steel GB								
Nom. length	*n = 8*	*Elim. CMI*	Nom. length	*n = 7*	*Elim. INMETRO*	Nom. length	*n = 6*	*STOP*
NMI	d_i	E_N	NMI	d_i	E_N	NMI	d_i	E_N
CMI	33.8	**1.9**	**INMETRO**	25.1	**1.4**	**NIST**	9.3	0.5
INMETRO	20.3	**1.2**	**INTI**	12.9	0.6	**INTI**	8.8	0.4
INTI	17.8	0.9	**NPLI**	10.9	0.5	**NPLI**	6.8	0.3
NPLI	15.8	0.7	**NIST**	5.1	0.3	**CEM**	3.8	0.2
CENAM	8.8	0.5	**CENAM**	3.9	0.2	**NRC**	2.3	0.1
NRC	6.8	0.2	**NRC**	1.9	0.1	**CENAM**	0.3	0.0
CEM	5.3	0.3	**CEM**	0.4	0.0	**CMI**	**42.8**	**2.0**
NIST	0.3	0.0				**INMETRO**	**29.3**	**1.4**

Table A3. Consecutive elimination of two inconsistent results to arrive to a set of 6 consistent ones for the 7 mm Steel GB.

10 mm, Steel GB					
Nom. length	*n = 8*	*Elim. CMI*	Nom. length	*n = 7*	*STOP*
NMI	d_i	E_N	NMI	d_i	E_N
CMI	23.3	**1.3**	**NPLI**	20.1	0.9
NPLI	16.8	0.7	**INTI**	12.9	0.6
INTI	16.2	0.8	**NRC**	9.9	0.4
NRC	13.2	0.5	**NIST**	8.1	0.4
CEM	11.2	0.6	**CEM**	7.9	0.4
NIST	4.8	0.3	**CENAM**	1.1	0.1
CENAM	2.2	0.1	**INMETRO**	1.1	0.1
INMETRO	2.2	0.1	**CMI**	**26.6**	**1.3**

Table A4. Consecutive elimination of one inconsistent result to arrive to a set of 7 consistent ones for the 10 mm Steel GB.

50 mm, Steel GB								
Nom. length	***n = 8***	***Elim. CEM***	**Nom. length**	***n = 7***	***Elim. CMI***	**Nom. length**	***n = 6***	***STOP***
NMI	d_i	E_N	**NMI**	d_i	E_N	**NMI**	d_i	E_N
CEM	33.1	**1.4**	**CMI**	21.1	0.9	**INTI**	18.8	0.6
CMI	25.9	**1.2**	**INTI**	22.4	0.8	**NPLI**	17.2	0.6
NPLI	18.4	0.6	**INMETRO**	18.4	0.6	**INMETRO**	14.8	0.5
INTI	17.6	0.6	**NPLI**	13.6	0.5	**NIST**	8.2	0.3
INMETRO	13.6	0.5	**NIST**	4.6	0.2	**NRC**	6.2	0.1
NIST	9.4	0.4	**NRC**	2.6	0.1	**CENAM**	2.2	0.1
NRC	7.4	0.2	**CENAM**	1.4	0.0	**CEM**	**34.3**	**1.2**
CENAM	3.4	0.1				**CMI**	**24.7**	0.9

Table A5. Consecutive elimination of two inconsistent results to arrive to a set of 6 consistent ones for the 50 mm Steel GB. In this case CMI was also eliminated even though their E_N Valueis smaller than 1, because they acknowledged they committed a mistake in the phase change correction.

75 mm, Steel GB											
Nom. length	***n = 8***	***Elim. NPLI***	**Nom. length**	***n = 7***	***Elim. CMI***	**Nom. length**	***n = 6***	***Elim. CEM***	**Nom. length**	***n = 5***	***STOP***
NMI	d_i	E_N	**NMI**	d_i	E_N	**NMI**	d_i	E_N	**NMI**	d_i	E_N
NPLI	112.3	**2.7**	**CMI**	64.3	**2.6**	**CEM**	42.0	**1.5**	**INTI**	13.4	0.4
CEM	68.8	**2.4**	**CEM**	52.7	**1.9**	**NRC**	19.0	0.6	**NRC**	10.6	0.4
CMI	48.3	**1.9**	**INTI**	15.7	0.4	**CENAM**	14.0	0.4	**INMETRO**	8.4	0.2
INTI	31.8	0.8	**INMETRO**	10.7	0.3	**NIST**	14.0	0.5	**CENAM**	5.6	0.2
INMETRO	26.8	0.7	**NRC**	8.3	0.3	**INTI**	5.0	0.1	**NIST**	5.6	0.2
CENAM	12.8	0.4	**CENAM**	3.3	0.1	**INMETRO**	0.0	0.0	**NPLI**	**130.6**	**2.7**
NIST	12.8	0.4	**NIST**	3.3	0.1				**CMI**	**66.6**	**2.3**
NRC	7.8	0.3							**CEM**	**50.4**	**1.6**

Table A6. Consecutive elimination of three inconsistent results to arrive to a set of 5 consistent ones for the 75 mm Steel GB.

100 mm, Steel GB								
Nom. length	*n = 8*	*Elim. CEM*	**Nom. length**	*n = 7*	*Elim. CMI*	**Nom. length**	*n = 6*	*STOP*
NMI	d_i	E_N	**NMI**	d_i	E_n	**NMI**	d_i	E_N
CEM	56.4	**1.7**	**CMI**	30.6	**1.0**	**NPLI**	15.7	0.3
CMI	38.6	**1.3**	**INMETRO**	19.4	0.4	**INMETRO**	14.3	0.3
NPLI	18.6	0.4	**CENAM**	12.4	0.3	**NRC**	8.7	0.3
NRC	11.6	0.4	**NPLI**	10.6	0.2	**CENAM**	7.3	0.2
INMETRO	11.4	0.2	**INTI**	10.4	0.2	**INTI**	5.3	0.1
NIST	5.6	0.2	**NRC**	3.6	0.1	**NIST**	2.7	0.1
CENAM	4.4	0.1	**NIST**	2.4	0.1	**CEM**	**59.3**	**1.5**
INTI	2.4	0.0				**CMI**	**35.7**	**1.0**

Table A7. Consecutive elimination of two inconsistent results to arrive to a set of 6 consistent ones for the 100 mm Steel GB.

1.000 5 mm, Ceramics GB		
Nom. length	*n = 8*	*STOP*
NMI	d_i	E_N
INTI	18.1	0.9
NRC	14.9	0.5
CENAM	10.1	0.5
NPLI	7.9	0.4
CEM	5.9	0.3
CMI	2.9	0.2
INMETRO	2.1	0.1
NIST	1.1	0.1

Table A8. All *8* results consistent for the 1.0005 mm Ceramics GB. No elimination needed.

5 mm, Ceramics GB		
Nom. length	*n = 8*	*STOP*
NMI	d_i	E_N
INTI	9.6	0.5
CEM	5.4	0.3
NRC	5.4	0.2
INMETRO	4.6	0.2
NIST	3.4	0.2
CMI	1.6	0.1
CENAM	1.4	0.1
NPLI	0.4	0.0

Table A9. All *8* results consistent for the 5 mm Ceramics GB. No elimination needed.

7 mm, Ceramics GB		
Nom. length	*n = 8*	*STOP*
NMI	d_i	E_N
INTI	15.7	0.8
NRC	14.3	0.5
INMETRO	8.7	0.4
NPLI	8.3	0.4
CEM	7.7	0.4
CENAM	5.3	0.3
NIST	2.3	0.1
CMI	1.8	0.1

Table A10. All *8* results consistent for the 7 mm Ceramics GB. No elimination needed.

10 mm, Ceramics GB		
Nom. length	*n = 8*	*STOP*
NMI	d_i	E_N
NRC	13.3	0.5
CEM	4.7	0.3
INMETRO	4.7	0.2
INTI	4.7	0.2
CMI	4.2	0.2
NIST	3.3	0.2
CENAM	1.3	0.1
NPLI	0.3	0.0

Table A11. All *8* results consistent for the 10 mm Ceramics GB. No elimination needed.

50 mm, Ceramics GB								
Nom. length	*n = 8*	*Elim. NPLI*	Nom. length	*n = 7*	*Elim. CENAM*	Nom. length	*n = 6*	*STOP*
NMI	d_i	E_N	NMI	d_i	E_n	NMI	d_i	E_N
NPLI	31.8	**1.1**	**CENAM**	28.4	**1.0**	**NRC**	24.1	0.9
CENAM	23.8	0.8	**CEM**	16.6	0.7	**INMETRO**	13.9	0.5
INMETRO	23.2	0.8	**CMI**	3.9	0.2	**CEM**	11.9	0.5
CEM	21.2	0.9	**NRC**	19.4	0.7	**INTI**	9.9	0.4
INTI	19.2	0.7	**NIST**	1.6	0.1	**CMI**	8.6	0.4
NRC	14.8	0.5	**INMETRO**	18.6	0.6	**NIST**	3.1	0.1
NIST	6.2	0.2	**INTI**	14.6	0.5	**NPLI**	**41.1**	**1.3**
CMI	0.7	0.0				**CENAM**	**33.1**	**1.1**

Table A12. Consecutive elimination of two inconsistent results to arrive to a set of 6 consistent ones for the 50 mm Ceramics GB.

75 mm, Ceramics GB					
Nom. length	***n = 8***	***Elim. CEM***	**Nom. length**	***n = 7***	***STOP***
NMI	d_i	E_N	**NMI**	d_i	E_N
CEM	68.8	**1.2**	**NPLI**	19.8	0.5
NPLI	48.3	0.6	**CENAM**	18.2	0.5
CMI	31.8	0.4	**NRC**	14.8	0.5
NRC	26.8	0.6	**INMETRO**	12.2	0.3
CENAM	13.5	0.4	**INTI**	10.2	0.3
NIST	12.8	0.2	**CMI**	4.3	0.2
INMETRO	12.8	0.2	**NIST**	1.8	0.1
INTI	**7.8**	0.2	**CEM**	**37.7**	**1.2**

Table A13. Consecutive elimination of one inconsistent result to arrive to a set of 7 consistent ones for the 75 mm Ceramics GB.

100 mm, Ceramics GB					
Nom. length	***n = 8***	***Elim. NPLI***	**Nom. length**	***n = 7***	***STOP***
NMI	d_i	E_N	**NMI**	d_i	E_N
NPLI	44.8	0.9	**CEM**	26.9	0.8
CEM	33.3	**1.0**	**CMI**	25.1	0.9
INMETRO	28.3	0.6	**NRC**	25.1	0.8
CMI	18.8	0.6	**INMETRO**	21.9	0.5
NRC	18.8	0.6	**CENAM**	10.9	0.3
CENAM	17.3	0.4	**NIST**	7.1	0.2
INTI	4.3	0.1	**INTI**	2.1	0.0
NIST	0.8	0.0	**NPLI**	**51.1**	0.9

Table A14. Consecutive elimination of one inconsistent result to arrive to a set of 7 consistent ones for the 100 mm Ceramics GB.

Annex B Calculation of Alternate Statistical Parameters.

Steel gauge blocks / Nominal length, mm							
Statistical estimator	**1.000 5**	**5**	**7**	**10**	**50**	**75**	**100**
Simple arithmetic mean	**-10.3**	**24.8**	**-5.3**	**35.9**	**4.8**	**-105.6**	**-43.7**
Standard uncertainty	**4.5**	**4.6**	**4.6**	**4.2**	**7.1**	**8.5**	**9.7**
Birge Ratio	**0.59**	**0.99**	**0.60**	**1.03**	**0.81**	**0.54**	**0.47**
Weighted mean	-8.8	25.9	-4.5	35.8	4.8	-103.5	-41.5
Standard uncertainty	4.2	4.3	4.4	4.0	6.5	8.2	8.8
Birge Ratio	0.61	1.12	0.64	1.08	0.88	0.53	0.45
Median	-11.8	21.5	-4.0	37.0	9.0	-100.0	-45.0
Observed chi-squared	1.9	6.3	2.0	5.6	3.8	1.1	1.0
Degrees of freedom	5	5	5	6	5	4	5
$Pr\{\chi^2(\nu) \geq \chi^2_{obs}\}$	0.866	0.281	0.842	0.471	0.572	0.888	0.962
Reduced chi-squared	0.37	1.25	0.41	0.93	0.77	0.28	0.20

Ceramics gauge blocks / Nominal length, mm							
Statistical estimator	**1.000 5**	**5**	**7**	**10**	**50**	**75**	**100**
Simple arithmetic mean	**-3.9**	**10.6**	**51.7**	**-14.3**	**105.9**	**136.2**	**-23.1**
Standard uncertainty	**3.9**	**4.0**	**4.0**	**4.1**	**5.6**	**6.9**	**7.8**
Birge Ratio	**0.95**	**0.46**	**0.89**	**0.54**	**1.07**	**0.78**	**1.02**
Weighted mean	-4.3	10.6	50.6	-15.3	106.2	137.3	-18.5
Standard uncertainty	3.7	3.8	3.8	3.9	5.4	6.5	7.0
Birge Ratio	0.89	0.47	0.84	0.48	1.05	0.73	1.21
Median	-3.0	11.5	53.8	-18.8	102.5	138.0	-21.0
Observed chi-squared	5.6	1.6	4.9	1.6	5.5	3.2	8.8
Degrees of freedom	7	7	7	7	5	6	6
$Pr\{\chi^2(\nu) \geq \chi^2_{obs}\}$	0.591	0.980	0.667	0.978	0.358	0.781	0.187
Reduced chi-squared	0.79	0.22	0.71	0.23	1.10	0.54	1.46

Steel gauge blocks with CMI correction/ Nominal length, mm							
Statistical estimator	**1.000 5**	**5**	**7**	**10**	**50**	**75**	**100**
Simple arithmetic mean	**-9.3**	**26.4**	**-2.6**	**36.2**	**4.9**	**-105.6**	**-42.0**
Standard uncertainty	**4.1**	**4.2**	**4.2**	**3.9**	**6.2**	**8.5**	**8.5**
Birge Ratio	**0.60**	**1.00**	**0.85**	**0.97**	**0.77**	**0.54**	**0.49**
Weighted mean	-7.9	27.7	-1.3	36.2	5.0	-103.5	-38.9
Standard uncertainty	3.9	3.9	4.0	3.7	5.6	8.2	7.6
Birge Ratio	0.60	1.10	0.92	0.90	0.80	0.53	0.47
Median	-10.5	25.0	-3.0	37.0	7.0	-100.0	-41.0
Observed chi-squared	2.1	7.2	5.1	5.7	3.8	1.1	1.30
Degrees of freedom	6	6	6	7	6	4	6
$Pr\{\chi^2(\nu) \geq \chi^2_{obs}\}$	0.907	0.301	0.534	0.580	0.697	0.888	0.970
Reduced chi-squared	0.36	1.20	0.85	0.81	0.64	0.28	0.22

Annex C Correspondence with Participants.

C.1 Correspondence with NMISA.

Dear Carlos
The gauge blocks were calibrated using a Tungsten Carbide platen for both the steel and ceramic gauge blocks. No Phase correction were calculated as the wring not of a good quality.

After a long investigation the decision at NMISA was to change back to quartz platens.

Because of this NMISA would like if at all possible to re-measure the gauge blocks using quartz platens?

Please let me know if this is possible?
Regards
Oelof

C.2 Correspondence with CMI.

Dear Carlos,

I saw the results and check it in my papers and I found out that I made big mistake in calculation of phase correction of steel gauge blocks. In the calculation I confused the deviation of the pack and sum of deviations of the n individual GBs. Than I had wrong result (+12nm) and the correct result should be (-12nm) – all deviations in central points of steel GB I sent you 24 higher than I should. At the time I didn´t check it properly because the results was similar to the results of comparison method and we had delay and I hurried.

For many years we didn´t have any problems during the measuring but when we started this comparison all was agains us. First we had problems with air-conditioning during the comparison measuring and than during the interferometry measuring broke down the cammera and than also the computer so all I had to calculate manually. Last year we did reconstruction of our NPL TESA interferometer in co-operation with Swiss METAS and now we have new camera, computer and software.

Is possible to repair my mistake in results or is too late for this?

I am very apoligize for my confusions!

Best regards

Frantisek

Czech metrology institute

Annex D Analysis of Results considering CMI correction.

Following are the recalculated reference values considering the correct correction of the phase change correction by CMI.

Comparison Reference Values			
Nominal length	Steel		
	Ref Val	$U_{Ref\ val}$	$n_{Ref\ Val}$
1.000 5	-9.3	8.2	7
5	26.4	8.3	7
7	-2.6	8.3	7
10	36.2	7.8	8
50	4.9	12.5	7
75	-105.6	17.0	5
100	-42.0	17.1	7

Table D1. Reference values (simple mean of largest sub-set of "consistent" results) deviations from Nominal Value with their Expanded Uncertainty and number of values contributing to the calculation of the Reference Value Computation (*n*) for both steel and ceramics GB.

NMI (*i*→)	CEM			CENAM			CMI			INMETRO		
Nom L (*j*↓)	d_{ij}	$U(d_{ij})$	E_{Nij}	d_{ij}	$U(d_{ij})$	E_{Nij}	d_{ij}	$U(d_{ij})$	E_{Nij}	d_{ij}	$U(d_{ij})$	E_{Nij}
1.005	-1.2	18.8	0.1	5.3	19.2	0.3	5.8	18.8	0.3	23.3	16.0	**1.3**
5	-9.9	19.0	0.5	8.6	19.2	0.5	9.6	18.8	0.5	21.6	18.0	**1.1**
7	1.1	-1.5	0.1	-2.4	-5.0	0.1	16.1	13.5	0.9	26.6	24.0	**1.3**
10	-8.2	19.2	0.4	0.8	19.6	0.0	2.3	19.0	0.1	0.8	18.0	0.0
50	-34.4	23.8	**1.3**	2.1	29.6	0.1	0.6	21.8	0.0	-14.9	32.0	0.5
75	-50.4	28.8	**1.6**	5.6	38.8	0.2	42.6	25.0	**1.5**	-8.4	44.0	0.2
100	-61.0	34.6	**1.6**	-9.0	49.0	0.2	10.0	29.0	0.3	-16.0	56.0	0.3

Table D2 A. Deviation from reference value for each GB, d_{ij}; claimed standard uncertainty, U_{ij}; and Normalized Error E_{Nij} of the **the Steel GB** for the first four participants.

NMI ($i\rightarrow$)	**INTI**			**NIST**			**NPLI**			**NRC**		
Nom L ($j\downarrow$)	d_{ij}	$U(d_{ij})$	E_{Nij}	d_{ij}	$U(d_{ij})$	E_{Nij}	d_{ij}	$U(d_{ij})$	E_{Nij}	d_{ij}	$U(d_{ij})$	E_{Nij}
1.005	-3.7	22.0	0.2	8.3	18.0	0.5	-5.7	22.0	0.3	-8.7	30.0	0.3
5	-13.4	22.0	0.7	14.6	18.8	0.8	-8.4	24.0	0.4	-1.4	30.0	0.1
7	-11.4	-14.0	0.6	6.6	4.0	0.4	-9.4	-12.0	0.4	-0.4	-3.0	0.0
10	-13.2	22.0	0.6	7.8	19.6	0.4	19.8	26.0	0.8	-10.2	30.0	0.4
50	-18.9	32.0	0.6	8.1	27.0	0.3	17.1	32.0	0.6	6.1	50.0	0.1
75	-13.4	42.0	0.4	5.6	31.4	0.2	130.6	46.0	**2.7**	10.6	32.0	0.4
100	-7.0	54.0	0.1	1.0	36.0	0.0	14.0	52.0	0.3	7.0	32.0	0.2

Table D2 B. Deviation from reference value for each GB, d_{ij}; claimed standard uncertainty, U_{ij}; and Normalized Error E_{Nij} of the **the Steel GB** for the last four participants.

Annex E Phase Change Correction Determination by the Stack Method.

A method usually applied to determine Δl_ϕ is the stack method where three or more GB are measured individually and then measured wrung together into a stack as shown in Figure 1. From these measurements the global phase change correction for this set of GB may be obtained as follows:

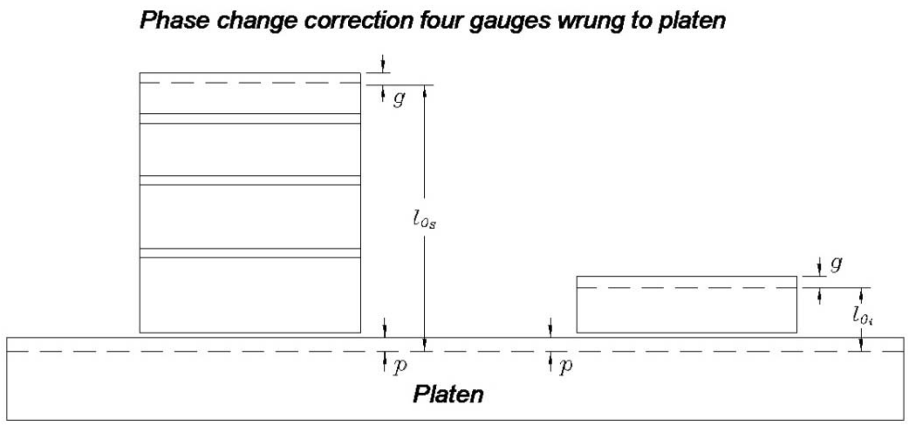

Figure 1. Stack method measurements to derive Δl_ϕ. g represents the difference between the optical plane and the mechanical plane of the GB free surface, and p represents this difference between planes for the platen.

$$\Delta l_\phi = \frac{l_{O_S} - \sum_{i=1}^{N} l_{O_i}}{N-1} \qquad \textbf{(E1)}$$

Where:

l_{Os} – Optical central length of the stack.

l_{Oi} – Optical central length of the i^{th} individual GB, $i = 1,2,\ldots,N,$ of the stack.

N – Number of gauge blocks in the stack.