Finding operating points in networks containing MOS transistors by a piecewise-linear approach

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Abstract— **In this paper we present a methodology for finding operating points in networks containing MOS transistors, linear positive resistors, and independent voltage and current sources. The MOS transistors are described by the high-canonical piecewise-linear (HC-PWL) model. A hybrid formulation is obtained from the network under analysis and it is solved in order to find the solution(s). The methodology takes advantage from the uniformly spaced simplicial partition in which the HC-PWL model is based because it makes possible to apply the Kuh-Chien algorithm. This algorithm has proved its efficiency to solve nonlinear resistive networks into simplicial subdivision schemes.**

Keywords— **DC solutions, Simplicial partition, High canonical piecewise-linear, MOS transistor networks**

I INTRODUCTION

The task of finding the operating points, in networks containing elements which are described by piecewise-linear (PWL) models, is a topic of interest in nonlinear circuit theory. Although numerous important references about PWL modeling and analysis can be found [1]- [4], there are not reported techniques which deal with the problem of applying the HC-PWL to DC analysis, and specifically with the task of computing operating points. This paper intends to be a first draft to overcome such problem. The main contribution of this paper is a methodology which is based on the Kuh-Chien algorithm [9], for computing operating points by a PWL approach where the nonlinear elements are described by the HC-PWL model. The network under study is considered to include MOS transistors which are described by the two dimensional HC-PWL model reported by Julián "*et al.*" in references [5] and [6]. Such model describes a n-dimensional PWL function $f(\mathbf{x})$ by the analytical expression

$$f(\mathbf{x}) = \mathbf{C}^T \mathbf{\Lambda}(\mathbf{x}), \forall \mathbf{x} \in S$$

Defined over a rectangular compact domain S in R^n

$$S = \{(x_1, \cdots, x_n): \ \ 0 \leq x_i \leq m_i\delta, \ \ i \in \{1, \cdots, n\}\}$$

when δ is a parameter called the grid step and m_i defines the rectangular length which is simplex partitioned.
$\mathbf{C}$ is denoted as vector of parameters and $\mathbf{\Lambda}$ is an expression which contains terms described by the so called absolute value γ function that is given by

$$\gamma(f_i, f_j) = \frac{1}{4}\{||-f_i| + f_j| - |-f_i + |f_j||\} + \frac{1}{4}\{|-f_i| + |f_j| - |-f_i + f_j|\}$$

with f_i and f_j as hyperplane equations.

A more detailed explanation about the HC-PWL model can be found in references [5], [6], and [7]. In this paper we are specifically interested in a methodology for computing operating points that is compatible with the HC-PWL model. The model description of the MOS transistors is assumed to be the explicit form of the HC-PWL representation reported in [7].

II MOS TRANSISTOR NETWORK

Fig.1 shows a 2μ-port network terminated by μ MOS transistors. In the network there are linear positive resistors and independent voltage and current sources.

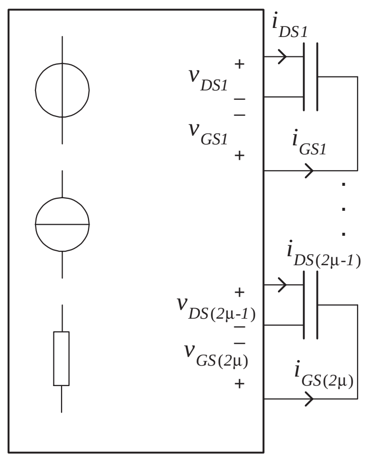

Figure 1: A 2μ port terminated by MOS transistors.

A hybrid formulation for the 2μ-port is reported in reference [8] and it is given by

$$\begin{bmatrix} \mathbf{i}_\alpha \\ \mathbf{v}_\beta \end{bmatrix} = -\begin{bmatrix} \mathbf{H}_{\alpha\alpha} & \mathbf{H}_{\alpha\beta} \\ \mathbf{H}_{\beta\alpha} & \mathbf{H}_{\beta\beta} \end{bmatrix} \begin{bmatrix} \mathbf{v}_\alpha \\ \mathbf{i}_\beta \end{bmatrix} + \begin{bmatrix} \mathbf{d}_\alpha \\ \mathbf{d}_\beta \end{bmatrix} \quad (1)$$

where

$$\mathbf{i}_\alpha = \left[i_{DS1} \cdots i_{DS(2\mu-1)}\right]^T, \mathbf{i}_\beta = \left[i_{GS2} \cdots i_{GS(2\mu)}\right]^T,$$
$$\mathbf{v}_\alpha = \left[v_{DS1} \cdots v_{DS(2\mu-1)}\right]^T, \mathbf{v}_\beta = \left[v_{GS2} \cdots v_{GS}(2\mu)\right]^T$$

are port currents and port voltages, $\mathbf{d}_\alpha$, $\mathbf{d}_\beta$ are subvectors of source vector.
From the MOS physics characteristics it can be seen that $\mathbf{i}_\alpha = \mathbf{f}(\mathbf{v}_\alpha, \mathbf{v}_\beta)$ and $\mathbf{i}_\beta = \mathbf{0}$. So that the hybrid formulation is reduced to:

$$\mathbf{f}(\mathbf{v}_\alpha, \mathbf{v}_\beta) + \mathbf{H}_{\alpha\alpha}\mathbf{v}_\alpha = \mathbf{d}_\alpha \quad (2)$$
$$\mathbf{H}_{\beta\alpha}\mathbf{v}_\alpha + \mathbf{v}_\beta = \mathbf{d}_\beta \quad (3)$$

III TWO DIMENSIONAL SIMPLICIAL SUBDIVISION

In the network shown in Fig.1, it is considered that the functions $i_\alpha = f(v_\alpha, v_\beta)$ are defined over a simplicial partition space. Hence, it is important to introduce the notation that here after will be used to refer the simplices. Figure 2 shows a two-dimensional space which has been simplically partitioned.

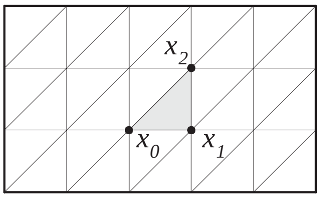

Figure 2: A two-dimensional simplicial partition.

Let $\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2$ be three points in this space.
A simplex $S(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2)$ is defined by

$$S = (\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2) = \left\{ \mathbf{x} : \mathbf{x} = \sum_{i=0}^{2} \mu_i \mathbf{x}_i \right\} \quad (4)$$

with $1 \geq \mu_i \geq 0$, $i = \{0, 1, 2\}$

and $\sum_{i=0}^{2} \mu_i = 1$

The points $\mathbf{x}_0, \mathbf{x}_1$, and $\mathbf{x}_2$ are called vertices of the simplex. Corresponding to three vertices, there are three boundaries B_k.
B_k contains all the vertices except $\mathbf{x}_k$ and it is defined as

$$B_k = \{\mathbf{x} : \mathbf{x} \in S(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2)\} \quad (5)$$

with $\mu_k = 0$, and $k = \{0, 1, 2\}$

The intersection of two boundaries is called corner. Thus every vertex is in fact a corner. Fig.3 depicts the geometrical relation between the vertices and boundaries for any simplex in a two-dimensional space.

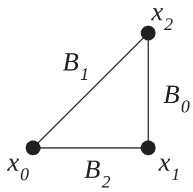

Figure 3: $S = (\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2)$ and its boundaries.

IV REPLACEMENT RULE

In reference [9], Kuh and Chien proposed a technique denominated replacement rule. It permits to trace a path in a structure of simplices. The technique considers that a new simplex can be reached by deleting a vertex and crossing its boundary. This idea is graphically depicted in Fig.4.

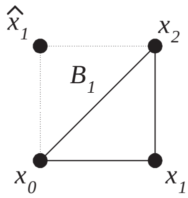

Figure 4: Replacement rule.

Let the boundaries set be defined by

$$B_k = \left\{ \mathbf{x} : \begin{bmatrix} \mathbf{x} \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{x}_0 & \mathbf{x}_1 & \mathbf{x}_2 \\ 1 & 1 & 1 \end{bmatrix} \right\} \mu \quad (6)$$

where the k-th component of μ is zero, ($\mu_k = 0$)

The new region is determined by the boundary B_k, and a new vertex $\hat{\mathbf{x}}_k$ is computed from

$$\hat{\mathbf{x}} = \mathbf{x}_{k+1} + \mathbf{x}_{k-1} - \mathbf{x}_k \quad (7)$$

with $k = 1$ at reference value

Equation (7) is usefull in the iterative process of finding an operating point, because it indicates how to traverse simplices until the solution is reached.

V THE KUH-CHIEN ALGORITHM

Let the two-dimensional equation $\mathbf{Y}(\mathbf{x}) = \mathbf{G}(\mathbf{x})$ be defined on a finite simplicial partition domain.
Let $S(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2)$ be any simplex in a simplicial partition. An affine function approximating the given $\mathbf{g}(\cdot)$ on $S(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2)$ can be defined by

$$\mathbf{Y}(\mathbf{x}) = [\mathbf{g}(\mathbf{x}_0), \mathbf{g}(\mathbf{x}_1), \mathbf{g}(\mathbf{x}_2)]\,\mu \quad (8)$$

for $x \in S(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2)$ and $\mu = [\mu_0, \mu_1, \mu_2]^T$
Let adopt the following representation:

$$\begin{bmatrix} \mathbf{Y}(\mathbf{x}) \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{g}(\mathbf{x}_0) & \mathbf{g}(\mathbf{x}_1) & \mathbf{g}(\mathbf{x}_2) \\ 1 & 1 & 1 \end{bmatrix} \mu \quad (9)$$

for $\mathbf{x} \in S(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2)$ with

$$\begin{bmatrix} \mathbf{x} \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{x}_0 & \mathbf{x}_1 & \mathbf{x}_2 \\ 1 & 1 & 1 \end{bmatrix} \mu \tag{10}$$

Then, equation (9) can be rewritten as

$$\begin{bmatrix} \mathbf{Y}(\mathbf{x}) \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{g}(\mathbf{x}_0) & \mathbf{g}(\mathbf{x}_1) & \mathbf{g}(\mathbf{x}_2) \\ 1 & 1 & 1 \end{bmatrix} \mathbf{X}_\mu \tag{11}$$

with

$$\mathbf{X}_\mu = \begin{bmatrix} \mathbf{x}_0 & \mathbf{x}_1 & \mathbf{x}_2 \\ 1 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{x} \\ 1 \end{bmatrix} \tag{12}$$

Since $\mathbf{x} \in S(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2)$ if and only if the vector μ satisfies the condition $1 \geq \mu \geq 0$, then it is very easy to check whether an approximate solution of $\mathbf{Y} = \mathbf{G}(\mathbf{x})$, is found in $S(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2)$ if and only if the solution of eq. (9) satisfies $\mu \geq \mathbf{0}$.
If there exists any negative element in μ, then the solution must be reached in other simplex. It implies an iterative process of solving eq.(9) and checking μ. Aided by the replacement rule, the new region which the solution enters is easily determined.

VI METHODOLOGY

Let a nonlinear network containing linear resistors, independent voltage sources, independent current sources and MOS transistors be described by the HC-PWL model as a the analytical PWL function $i_{DS} = \mathbf{C}^T \mathbf{\Lambda}(v_{DS}, v_{GS})$. The methodology for finding operating points is summarized in the following steps:

- *Step 1:* Obtain the reduced hybrid formulation for the network depicted in Fig. 1 as

$$\mathbf{f}(\mathbf{v}_\alpha, \mathbf{v}_\beta) + \mathbf{H}_{\alpha\alpha}\mathbf{v}_\alpha = \mathbf{d}_\alpha \tag{13}$$
$$\mathbf{H}_{\beta\alpha}\mathbf{v}_\alpha + \mathbf{v}_\beta = \mathbf{d}_\beta \tag{14}$$

  where $\mathbf{f}(\mathbf{v}_\alpha, \mathbf{v}_\beta)$ has the form $\mathbf{C}^T \mathbf{\Lambda}(v_{DS}, v_{GS})$

- *Step 2:* Recast the reduced hybrid formulation into the form $\mathbf{Y} = \mathbf{0}$ as

$$\mathbf{Y} = \begin{bmatrix} \mathbf{f}(\mathbf{v}_\alpha, \mathbf{v}_\beta) + \mathbf{H}_{\alpha\alpha}\mathbf{v}_\alpha - \mathbf{d}_\alpha \\ \mathbf{H}_{\beta\alpha}\mathbf{v}_\alpha + \mathbf{v}_\beta - \mathbf{d}_\beta \end{bmatrix} = \mathbf{0} \tag{15}$$

- *Step 3:* Start with the simplex: $\{\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2\}$
- *Step 4:* Evaluate the simplex vertices into the system $\mathbf{Y}(\mathbf{x}) = \mathbf{G}(\mathbf{x}) = \mathbf{0}$

$$\mathbf{Y}(\mathbf{x}) = [\mathbf{g}(\mathbf{x}_0), \mathbf{g}(\mathbf{x}_1), \mathbf{g}(\mathbf{x}_2)] = \mathbf{0} \tag{16}$$

- *Step 5:* Form the system for the i-th iteration

$$\begin{bmatrix} \mathbf{Y}(\mathbf{x}) \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{G}(\mathbf{x}) \\ 1 \end{bmatrix} \mu^{(\mathbf{i})} = \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} \tag{17}$$

  with

$$\mu^{(\mathbf{i})} = \begin{bmatrix} \mu_0^{(i)} \\ \mu_1^{(i)} \\ \mu_2^{(i)} \end{bmatrix} = \begin{bmatrix} \mu_0^{(i)}, & \cdots & , \mu_k^{(i)} \end{bmatrix}^T \tag{18}$$

  and $k \in \{0, 1, 2\}$

- *Step 6* Solve equation (17) for $\mu^{(\mathbf{i})}$
- *Step 7:* Search the k-th element of $\mu^{(\mathbf{i})}$ which is negative. If all the elements of $\mu^{(\mathbf{i})}$ are positive, then go to *Step 9,* otherwise go to *Step 8*
- *Step 8:* Apply the replacement rule to the $\mathbf{x}_k$ vertex and return to *Step 3*

$$\tilde{\mathbf{x}}_k = \mathbf{x}_{k+1} + \mathbf{x}_{k-1} - \mathbf{x}_k \tag{19}$$

- *Step 9:* The solution is found. Compute the solution by

$$[\mathbf{x}] = \begin{bmatrix} \mathbf{x}_0 & \mathbf{x}_1 & \mathbf{x}_2 \end{bmatrix} \mu^{(\mathbf{i})} \tag{20}$$

The above methodology is applied repeatedly in order to compute various operating points.

VII CASE STUDY

The latch circuit shown in Fig.5 is a well known three operating points circuit. We are interested in computing these solutions by applying the simplicial methodology presented in the previous section.
This circuit contains two nMOS transistors, two linear resistors (R_1 and R_2) and a voltage source (V_{DD}).

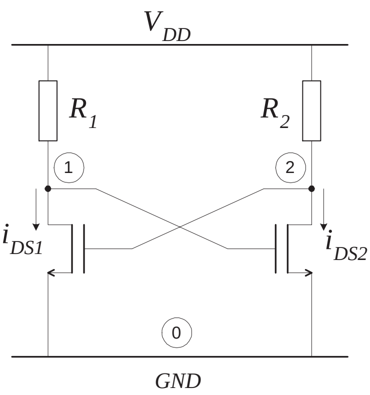

Figure 5: Example circuit.

The nMOS transistors are described by the two dimensional HC-PWL model. In order to make more legible the HC-PWL formulation for i_{DS}, the following notation is introduced

$$\begin{aligned} \delta_a^b &= a\,|v_{DS} - b| \\ \phi_a^b &= a\,|v_{GS} - b| \\ \gamma_a^b &= a\,|v_{DS} - v_{GS} + b| \\ \lambda_a^{b,c} &= a\,||v_{GS} - b| + v_{DS} - c| \\ &\quad - a\,|-v_{GS} + b + |v_{DS} - c|| \end{aligned}$$

And i_{DS} is given by

$$i_{DS} = \frac{1}{4}\{H_{PWL}\} \quad \mu A$$

where

$$\begin{aligned}H_{PWL} &= \delta_{32}^{3} + \delta_{125}^{0} + \delta_{-24}^{4} + \delta_{12}^{1} + \delta_{-14}^{2} + \phi_{12}^{4} + \phi_{28}^{3} \\ &+\phi_{57}^{2} + \lambda_{1}^{1,1} + \lambda_{-13}^{3,2} + \lambda_{-25}^{4,4} + \lambda_{-26}^{3,1} + \lambda_{9}^{2,1} + \lambda_{-50}^{2,0} \\ &+\lambda_{43}^{3,0} + \lambda_{23}^{3,3} + \lambda_{2}^{4,0} + \lambda_{30}^{1,0} + \lambda_{28}^{4,1} + \lambda_{1}^{1,4} + \lambda_{9}^{4,3} \\ &+\lambda_{-2}^{4,2} + \lambda_{1}^{1,3} + \lambda_{-1}^{2,3} + \lambda_{1}^{3,4} + \lambda_{-1}^{2,4} + \lambda_{1}^{1,2} + \lambda_{50}^{2,0}\end{aligned}$$

The piecewise linear function i_{DS} is described over an uniformly spaced grid within a finite rectangular region defined by $0 \leq v_{DS} \leq 5$ and $0 \leq v_{GS} \leq 5$ as shown in Fig.6. The grid $v_{DS} - v_{GS}$ is divided into a set of $1V$ squares and it is subdivided into a simplicial partition.

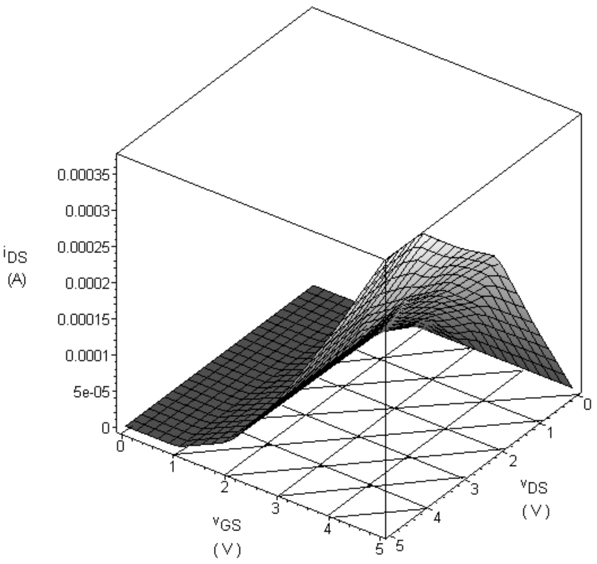

Figure 6: i_{DS} piecewise-linear curve.

The data points in Fig.6 follow the Shichman-Hodges model [10]; namely

$$i_{DS} = \tilde{K}\left[\left(v_{GS} - V_t\right) v_{DS} - \frac{1}{2} v_{DS}^2\right]$$

if $v_{DS} \leq v_{GS} - V_t$; or

$$i_{DS} = \frac{1}{2}\tilde{K}\left(v_{GS} - V_t\right)^2\left[1 + \tilde{\lambda}\left(v_{DS} - v_{GS} + V_t\right)\right]$$

if $v_{DS} > v_{GS} - V_t$, with $\tilde{K} = 50\mu A/V^2$, $V_t = 1V$ and $\tilde{\lambda} = 0.0V^{-1}$.
From Fig.5, the following nodal information is collected

$$node1: i_{DS1} + \frac{v_{DS1} - V_{DD}}{R_1} = 0, \quad v_{DS1} = v_{GS2} \qquad (21)$$

$$node2: i_{DS2} + \frac{v_{DS2} - V_{DD}}{R_2} = 0, \quad v_{DS2} = v_{GS1} \qquad (22)$$

Because of $i_{DS1} = f(v_{DS1}, v_{GS1})$ and $i_{DS2} = f(v_{DS2}, v_{GS2})$, then the above equations can be rewritten into the reduced hybrid system format as

$$f\left(v_{DS1}, v_{GS1}\right) + \left(\frac{1}{R_1}\right) v_{DS1} = \frac{V_{DD}}{R_1} \qquad (23)$$

$$\begin{aligned}f\left(v_{DS2}, v_{GS2}\right) + \left(\frac{1}{R_2}\right) v_{DS2} &= \frac{V_{DD}}{R_2} \qquad (24)\\ v_{DS1} - v_{GS2} &= 0 \\ v_{DS2} - v_{GS1} &= 0\end{aligned}$$

Notice that equation (23) and equation (24) can be recast as

$$f\left(e_1, e_2\right) + \left(\frac{1}{R_1}\right) e_1 = \frac{V_{DD}}{R_1} \qquad (25)$$

$$f\left(e_2, e_1\right) + \left(\frac{1}{R_2}\right) e_2 = \frac{V_{DD}}{R_2} \qquad (26)$$

where e_1 and e_2 are nodal voltages.

From equation (25) and equation (26), it can be defined the equation system $\mathbf{Y} = \mathbf{0}$ as

$$y_1 = i_{DS}\left(e_1, e_2\right) + \left(\frac{e_1 - V_{DD}}{R_1}\right) = 0 \qquad (27)$$

$$y_2 = i_{DS}\left(e_2, e_1\right) + \left(\frac{e_2 - V_{DD}}{R_2}\right) = 0 \qquad (28)$$

Let $R_1 = R_2 = 30K\Omega$ and $V_{DD} = 3.3V$. The solutions of the system $\mathbf{Y} = \mathbf{0}$ are computed as follows.
Firstly, a start simplex is chosen.
Let

$$x_0 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}, \; x_1 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \; x_2 = \begin{bmatrix} 3 \\ 5 \end{bmatrix} \qquad (29)$$

The equation to be solved at the first iteration is

$$\begin{bmatrix} g_1(\mathbf{x}_0) & g_1(\mathbf{x}_1) & g_1(\mathbf{x}_2) \\ g_2(\mathbf{x}_0) & g_2(\mathbf{x}_1) & g_2(\mathbf{x}_2) \\ 1 & 1 & 1 \end{bmatrix} \mu^{(\mathbf{i})} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad (30)$$

After substituting it yields

$$\begin{bmatrix} \mathbf{G}\left(\mathbf{x}\right) \\ 1 \end{bmatrix} \mu^{(\mathbf{0})} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad (31)$$

$$\mathbf{G}\left(\mathbf{x}\right) = \begin{bmatrix} 14.37 & 21.67 & 30 \\ 5.63 & 9.09 & 18.06 \end{bmatrix} \times 10^{-5} \qquad (32)$$

The solution is

$$\mu^{(\mathbf{0})} = \begin{bmatrix} \mu_0^{(0)} \\ \mu_1^{(0)} \\ \mu_2^{(0)} \end{bmatrix} = \begin{bmatrix} 3.22 \\ -2.44 \\ 0.22 \end{bmatrix} \qquad (33)$$

Since μ_1^0 is negative, then the vertex x_1 must be replaced by

$$\begin{bmatrix} 2 \\ 4 \end{bmatrix} + \begin{bmatrix} 3 \\ 5 \end{bmatrix} - \begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \qquad (34)$$

The new simplex is defined by

$$\begin{bmatrix} 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

In this simplex, the new equation to solve is

$$\begin{bmatrix} \mathbf{G}(\mathbf{x}) \\ 1 \end{bmatrix} \mu^{(\mathbf{1})} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{35}$$

$$\mathbf{G}(\mathbf{x}) = \begin{bmatrix} 14.37 & 20.5 & 30 \\ 5.63 & 14.6 & 18.06 \end{bmatrix} \times 10^{-5} \tag{36}$$

The solution is

$$\mu^{(\mathbf{1})} = \begin{bmatrix} \mu_0^{(1)} \\ \mu_1^{(1)} \\ \mu_2^{(1)} \end{bmatrix} = \begin{bmatrix} 1.065 \\ 1.405 \\ -1.47 \end{bmatrix} \tag{37}$$

And the vertex to be replaced is x_2 by $[2\ 3]^T$. The new simplex is then defined by

$$\begin{bmatrix} 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

The results in the third, fourth, and fifth iteration are

$$\mu^{(\mathbf{2})} = \begin{bmatrix} \mu_0^{(2)} \\ \mu_1^{(2)} \\ \mu_2^{(2)} \end{bmatrix} = \begin{bmatrix} -1.709 \\ 0.295 \\ 2.414 \end{bmatrix} \tag{38}$$

with the following simplex defined in

$$\begin{bmatrix} 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\mu^{(\mathbf{3})} = \begin{bmatrix} \mu_0^{(3)} \\ \mu_1^{(3)} \\ \mu_2^{(3)} \end{bmatrix} = \begin{bmatrix} 0.93 \\ -0.855 \\ 0.918 \end{bmatrix} \tag{39}$$

and the new simplex defined in

$$\begin{bmatrix} 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\mu^{(\mathbf{4})} = \begin{bmatrix} \mu_0^{(4)} \\ \mu_1^{(4)} \\ \mu_2^{(4)} \end{bmatrix} = \begin{bmatrix} 0.099 \\ 0.9 \\ 0 \end{bmatrix} \tag{40}$$

Since $\mu^{\mathbf{4}} > \mathbf{0}$, there is not following simplex to jump, then the solution is computed by

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 2 \\ 3 & 2 & 3 \end{bmatrix} \mu^{(\mathbf{4})} = \begin{bmatrix} 2.099 \\ 2.099 \end{bmatrix} \tag{41}$$

After applying the same algorithm into two different start simplices, the following solutions can be computed

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 3 & 3 & 4 \end{bmatrix} \mu^{(\mathbf{4})} = \begin{bmatrix} 0.857 \\ 3.3 \end{bmatrix} \tag{42}$$

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} 3 & 3 & 4 \\ 0 & 1 & 1 \end{bmatrix} \mu^{(\mathbf{5})} = \begin{bmatrix} 3.3 \\ 0.857 \end{bmatrix} \tag{43}$$

Notice that the three computed solutions can be substituted into the eq.(25) and eq.(26) and the condition $\mathbf{Y} = \mathbf{0}$ is fullfil. The paths followed by the algorithm to obtain the solutions are depicted in Fig.7.

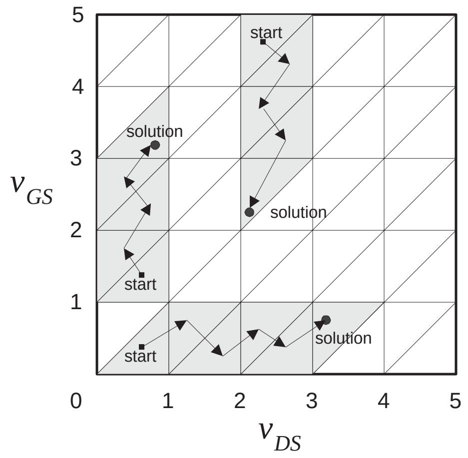

Figure 7: Path solutions into the simplicial partition.

VIII Conclusions

A methodology for finding operating points in networks containing MOS transistors was proposed. Such methodology is based into the Kuh-Chien algorithm and it is able to handle equation systems which involve the HC-PWL description. Because the algorithm is able to compute only one solution, in multiple operating point systems it is necessary to apply it as many times as solutions exists. It presents two important challenges, the first consists in determining previously to the analysis, the maximum number of existing solutions of the system. The other one consists in determining the optimal starting point for running the Kuh-Chien algorithm. The analysis of both problems promise interesting results in this topic under investigation

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