Development and validation of a least squares algorithm for static and dynamic synchrophasor tests

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Abstract: INTI had developed a least squares fit algorithm based on an iterative matrix resolution method, covering all tests of the IEEE C37.118.1 standard. We present the method, results, validation and conclusions of this work.

Keywords: Synchrophasor-PMU- Calibration-Algorithm –Least square.

1. INTRODUCTION

INTI is part of a public private consortium[1] which aim is to develop a measuring system to monitor the electrical grid, based on synchrophasors technology. A work packaged in this project includes the development of a reference system for PMU calibration. In this work we present an algorithm developed with that purpose, both for static and dynamic tests based on [1,2,3,4]

In Fig. 1 we can appreciate the block diagram of INTI PMU calibration system. A 120V and 5 A signal is applied to the Device under test (DUT) and to the digitalization stage (DAQ) via current and voltage transducers. The digitalized output signals from the DAQ are fitting using an algorithm based on the least square method (LS) for obtain the reference synchrophasors, and thus be able to compare them with those produced by the PMU under test

2. LS FITTING ALGORITHM

The signal injected by the reference signal source can be modeled as

$$V(\bar{t}) = V_0(\bar{t})\,.sen\,(2\pi\bar{t}\,.f(\bar{t}) + \varphi_0) + B \qquad (1)$$

where $V_0(\bar{t})$ is the amplitude of the signal, $f(\bar{t})$ is the frequency of the system, φ_0 is the phase angle and B is the DC component. The $\bar{t}$ vector represents the time of each sample.

The polynomial used by the fiting algorithm to estimate the reference signal can be seen in eq. (2) and eq. (3). Amplitude, frequency, phase and DC component are estimated using polynomials for each of them ranging from degree zero to degree two.

$$V_0(\bar{t}) = V_0 + V_1\,.\bar{t} + V_2\,.\bar{t}^{\,2} \qquad (2)$$

$$f(\bar{t}) = f_0 + f_1\,.\bar{t} \qquad (3)$$

The algorithm estimates $V_0, V_1, V_2, f_0, f_1, \varphi_0, B$.

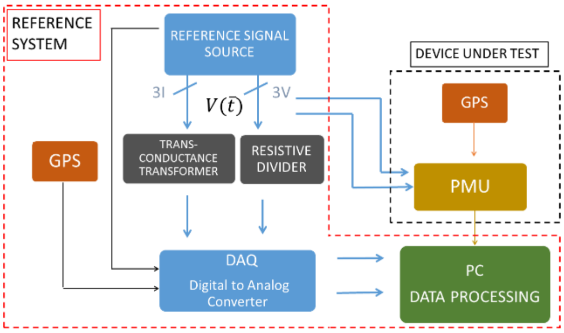


Figure 1. Scheme of INTI PMU calibration system.

We use expansion in Taylor series over $V(\bar{t})$ in function of frequency $f(\bar{t})$. To estimate the digitalized signal in the sample vector $\bar{v}$, we use:

[1] FONARSEC Project is financed by Ministerio de Ciencia, Tecnología e Innovación Productiva.

$$\bar{v} \cong H(\bar{t}, \bar{f}') \,.\, SC^t \tag{4}$$

where $\bar{f}'$ are the seed values of $f(\bar{t})$,

$$H(\bar{t}, \bar{f}') = \\ = [\, X(\bar{t}, \bar{f}') \;\; Y(\bar{t}, \bar{f}') \;\; \bar{t}\,.X(\bar{t}, \bar{f}') \;\; \bar{t}.Y(\bar{t}, \bar{f}') \\ \bar{t}^2.X(\bar{t}, \bar{f}') \;\; \bar{t}^2\,.Y(\bar{t}, \bar{f}') \;\; 1\,] \tag{5}$$

where

$$X(\bar{t}, \bar{f}') = sen\,(2\pi\bar{t}\,(\,f_0{}' + f_1{}'\,\bar{t}\,)) \tag{6}$$

$$Y(\bar{t}, \bar{f}') = cos\,(2\pi\bar{t}\,(\,f_0{}' + f_1{}'\,\bar{t}\,)) \tag{7}$$

and SC is the fit coefficients vector

$$SC = [\, S_0 \;\; C_0 \;\; S_1 \;\; C_1 \;\; S_2 \;\; C_2 \;\; C_3\,] \tag{8}$$

that minimizes the cuadratic vector error,

$$\left\| \bar{v} - H(\bar{t}, \bar{f}') \,.\, SC^t \right\|_2^2 \tag{9}$$

and we obtain the solution through the following equation:

$$SC(\bar{t}, \bar{f}') = [H^t(\bar{t}, \bar{f}')\,.H(\bar{t}, \bar{f}')]^{-1}.\, H^t(\bar{t}, \bar{f}')\,.\bar{v} \tag{10}$$

To solve (10) we use an iterative method, starting with an approximate initial value of $f_0{}'$ y $f_1{}'$. Then,

$$\Delta f_0 = (S_0\, C_1 - C_0 S_1)/(2\pi V_0^2) \tag{11}$$

$$\Delta f_1 = (S_0\, C_2 - C_0 S_2)/(2\pi V_0^2) \tag{12}$$

$$f_0 \cong f_0' + \Delta f_0 \tag{13}$$

$$f_1 \cong f_1' + \Delta f_1 \tag{14}$$

Once we obtain f_0 y f_1, we directly calculate

$$V_0 = \sqrt{S_0{}^2 + C_0{}^2} \tag{15}$$

$$V_i = (C_0\, C_i - S_0 S_i)/V_0 \tag{16}$$

$$\varphi_0 = arctg\left(\frac{C_0}{S_0}\right) \tag{17}$$

$$B = C_3 \tag{18}$$

3. COVERED TESTS

The developed method allows to perform all the tests that requires the cited standard of synchrophasors, either in voltage and current magnitudes. In all tests are calculated TVE (Total Vector Error), FE (Frequency Error) and RFE (ROCOF Frequency Error) [5]

Stationary Tests:

a. Magnitude and phase.
b. Harmonic distortion.
c. Interarmonic distortion.

Dynamic Tests:

d. Magnitude and phase modulation.
e. Frequency ramp.
f. Magnitude and phase step.

4. VALIDATION AND RESULTS

The validation of the algorithm was done through a comparison with METAS. The validation method consisted in the comparison of the results of the algorithms developed by both institutes, for stationary and dynamic tests.
In this paper only results for dynamic tests are presented.

4.1 FREQUENCY RAMP

The test consists of the application of a frequency ramp signal to the system that can be model as

$$X = X_m\, cos[\omega_0\, t + \pi R_f t^2] \tag{19}$$

where ω_0 is the nominal frequency of the system, and $R_f = \frac{df}{dt}$ is the ramp slope in Hz/s. For the test we used frequencies from *45* to *55* Hz with a rising of $R_f = 1\,Hz/s$, sampling frequency $f_s = 18kHz$ and $\omega_0 = 50\,Hz$. Table 1 and Figure 2 shows the results for TVE, FE y RFE

	DYNAMIC FREQ RAMP TEST				
	max	mean	min	max C37.118	passed
TVE (%)	0,08	0,04	0,001	1	✓
FE (Hz)	0,004	0,0036	0,0031	0,005	✓
RFE (Hz/s)	3x10^{-6}	3x10^{-6}	3x10^{-6}	0,1	✓

Table 1. Results of comparison between METAS and INTI for frequency ramp tests.

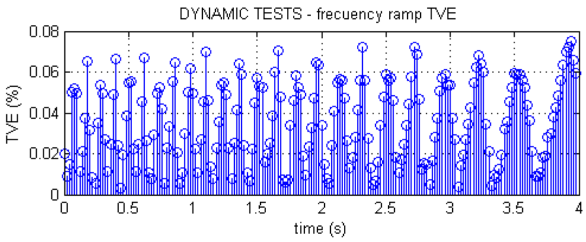


Figure 2. Frequency ramp TVE results.

4.2 AMPLITUDE AND PHASE MODULATION

The test consists of the application of a modulation signal to the system that can be model as,

$$X = X_m\,[1 + K_x\,cos(\omega\,t)].\cos[\omega_0\,t + K_a\,cos(\,\omega t - \pi)] \qquad (20)$$

where $X_m$is the amplitude of the signal, ω_0 is the nominal frequency of the system, ω is the modulation frequency, K_x is the amplitude modulation factor and K_a is the phase modulation factor.

4.2.1 AMPLITUDE MODULATION

For amplitude modulation test we used the following configuration: ω =0.1 to 4.9 Hz with gaps of 0.2 Hz. $K_x = 0.1$, $K_a = 0$, sampling frequency $f_s = 18$ kHz and $\omega_0 = 50$ Hz.
In table 2 and figures 3 and 4 we can see the results for TVE, FE y RFE.

	DYNAMIC AMPLITUDE MODULATION TEST				
	max	mean	min	max C37.118	passed
TVE (%)	0,075	0,033	0,001	3	✓
FE (Hz)	0,015	0,006	0,0001	0,06	✓
RFE (Hz/s)	0,33	0,19	0,002	3	✓

Table 2. Results of comparison between METAS and INTI for amplitude modulation tests.

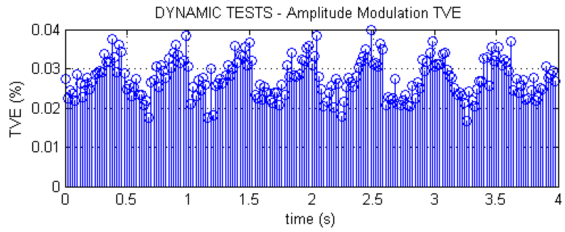


Figure 3. Amplitude modulation TVE results.

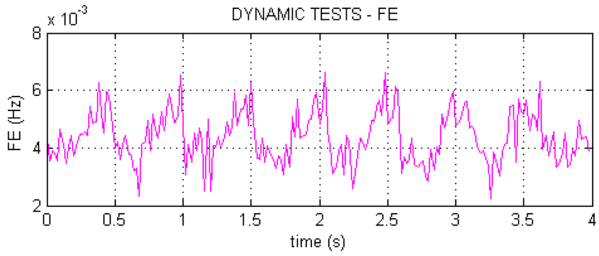


Figure 4. Amplitude modulation FE results.

4.2.2 PHASE MODULATION

For phase modulation a signals with the following configuration was applied: ω =0.1 to 4.9 Hz with steps of 0.2 Hz. $K_a = 0.1$, $K_x = 0$
In table 3 we can see the results for TVE, FE y RFE.

	DYNAMIC PHASE MODULATION TEST				
	max	mean	min	max C37.118	passed
TVE (%)	0,073	0,057	0,043	3	✓
FE (Hz)	0,05	0,03	0,0001	0,06	✓
RFE (Hz/s)	2,61	1,63	0,02	3	✓

Table 3. Results of comparison between METAS and INTI for phase modulation tests.

4.3 PHASE STEP

The mathematical representation of the applied signal is:

$$X = X_m\,[1 + K_x f_1(t)\,].\cos[\omega_0 t + K_a f_1(t)\,] \qquad (21)$$

where $X_m$is the amplitude of the signal, ω_0 is the nominal frequency of the system, $f_1(t)$ is a unit step function, K_x is the magnitude step size and K_a the phase step size. The sampling frequency $f_s = 18$ kHz.

The results presented below have been performed under the following conditions: number of tests: 180. Phase increase per test: $\frac{\pi}{90}\frac{\text{rad}}{\text{step}}$. K_x=1, $K_a = \frac{\pi}{90}\frac{\text{rad}}{\text{step}}$. Synchrophasor report rate: 50 $\frac{\text{frames}}{\text{s}}$. Step instant: 0.5 s

	DYNAMIC PHASE STEP TEST		
	max	max C37.118	passed
Response time (s)	0,038	0,199	✓
Delay time (s)	0,0023	0,005	✓
Overshoot (%)	1,8	5	✓

Table 4. Results of comparison between METAS and INTI for Phase step tests.

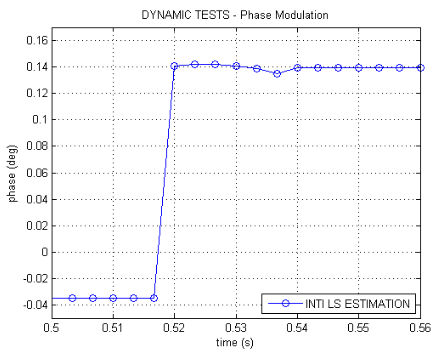
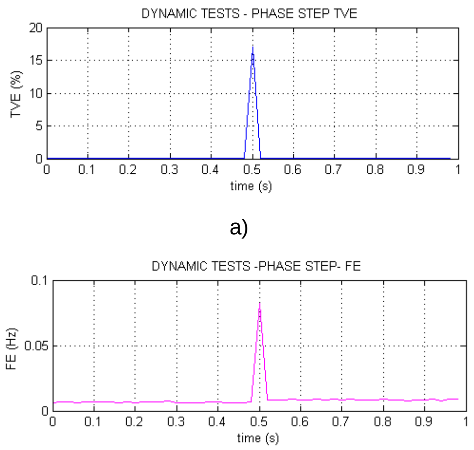


Figure 5. INTI estimation algorithm phase step fitting.



a)

b)

Figure 6. Phase step results referred to: a) Response time. b) Delay.

5. CONCLUSIONS

Iterative matrix adjustment algorithms have been developed, using the Taylor series expansion method. The algorithms have been validated and the obtaining results in all tests were below the required limits by the synchrophasor standard IEEE C37.118.1. We can concluded that the algorithm developed at INTI can be used in the PMU reference system

We are currently working on the development of algorithms based on other methods to compare and optimize each tests.

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