UNCERTAINTY EVALUATION IN A TWO-TERMINAL CRYOGENIC CURRENT COMPARATOR

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Abstract

In this paper we present the uncertainty evaluation of a new cryogenic current comparator (CCC) bridge designed to compare two-terminal 1 MΩ and 10 MΩ standard resistors with the quantized Hall resistance (QHR) and then scale from these values to other values between 10 kΩ and 1 GΩ.

Introduction

The CCC is used in many national metrology laboratories to obtain high accuracy four-terminal resistance measurements in the range of 1 Ω to 10 kΩ. Some papers show that this traditional CCC can be used to measure high value resistors with low uncertainty [1]. Here we analyze a two-terminal CCC that does not use a voltage detector and requires only a single voltage source, making it much easier to use than the traditional CCC. This type of bridge has been used at NIST to compare the QHR directly with 1 MΩ and 100 MΩ [2] resistors. The recent two-terminal CCCs [3] developed to compare the QHR with room-temperature resistors use resistive windings. This paper presents a complete study of the uncertainty in the new CCC which provides new capabilities in resistance scaling for high-value resistors.

Two-terminal resistive-winding CCC

Figure 1 shows a schematic diagram of the two-terminal CCC. A source voltage is applied directly to the two resistors under test, in parallel. Each resistor is in series with a winding, and the two windings have opposite directions. A superconducting quantum interference device (SQUID) senses the total flux and drives a feedback winding of one turn to maintain the flux balance in the bridge. This bridge uses six major windings. The four-turn winding is superconducting and is used with the QHR. The 3100-turn, 310-turn and 31-turn windings are made of phosphor-bronze wire and these have nominal resistances at 4.2 K of 2500 Ω, 250 Ω and 25 Ω, respectively. The internal resistances of the windings decrease the effect of self-resonance in the CCC. This improves the sensitivity [2] but the winding resistance must be measured to correct the value of each resistor. Since this bridge measures two-terminal resistors, we use the triple-series connection technique to reduce errors in measurements of the QHR, as described in [4]. In a condition of balance, the bridge equations are:

$$I_1 N_1 = I_2 N_2 + I_F N_F + \Delta, \tag{1}$$

$$I_1 = \frac{V}{R_1 + r_{w1}}, \quad I_2 = \frac{V}{R_2 + r_{w2}}. \tag{2}$$

Here, I_j is the current in the resistor j, r_{wj} is the resistance in the connections of the j bridge arm, N_j is the number of turns in the winding j, V is the source voltage, I_F is the feedback current and Δ includes all systematic contributions.

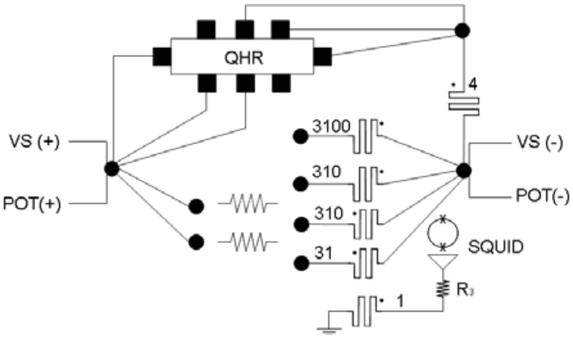

Figure 1. Schematic diagram of the bridge, showing connections for room-temperature resistors and the voltage source (VS).

Uncertainty components

Type A evaluation of standard uncertainty

First, we describe components which produce random errors in the result and can be eliminated by averaging or reduced by a correct design of the system.

- **Johnson-Nyquist noise.** The Johnson noise of the resistor R_j is estimated from the flux produced by the Johnson current which flows in the winding j.
- **SQUID noise.** The SQUID produces 1/f and white noise components. The first is reduced by alternating the current polarity and by averaging. The white noise is specified by the manufacturer in units of $\mu\Phi_0/\mathrm{Hz}^{1/2}$ where Φ_0 is the flux quantum.
- **Electromagnetic interference and vibration noise.** These types of interference are very difficult to estimate but a good design can be used to eliminate these effects. The CCC is surrounded by different levels of shielding. All the wires connecting the CCC with the electronics or the resistor are shielded and twisted. Inside the CCC the interconnecting leads are twisted together and fixed rigidly to the CCC probe.
- **Noise in CCC electronics: voltage source, feedback current and feedback sense.** The voltage source and the feedback current source were designed

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using low pass filters and low noise components. To measure the feedback current we use a sense resistor and two buffers, one at each resistor terminal. These buffers produce high frequency voltage noise and 1/f noise due to the temperature dependence of the voltage offset.

Type B evaluation of standard uncertainty

These components produce systematic offsets in the result.

- **Winding and lead resistance.** The uncertainty produced by the correction of the winding and lead resistance has three components: measurement device, current coefficient and the dependence on helium level. The first two are the most important because the last can be eliminated if the resistances are frequently measured.
- **Measurement of the feedback current.** This is calculated as the uncertainty produced by the multimeter and the uncertainty in the calibration of the sense resistor.
- **Voltage source and thermal EMF.** This bridge does not use a feedback loop to balance the voltage across each resistor, so the stability of the source produces an uncertainty in the result. For the same reason, the voltage across each resistor can be affected by the resistance of the winding or leads and the thermal electromotive forces (EMF). The first can be corrected and the second can be eliminated using the voltage reversal measurement technique, if the thermal EMF is constant.
- **Leakage current.** Using the guard technique we can eliminate the possibility of leakage in parallel to the resistors, which can exist especially in Hamon devices. Leakage current can affect only the positive terminals of the windings, and produces an estimated error of 0.02 μΩ/Ω in 100 MΩ.
- **CCC current-linkage error.** With effective shielding, reversing the current passing through two windings of equal number in series-opposition should produce no change in the voltage output of the SQUID, when it is not connected to the feedback circuit**.**
- **SQUID feedback null.** The external feedback must maintain constant output voltage in the SQUID electronics. If it is different from zero and is not constant it will produce an error.
- **Resistor time constant.** The resistor under test should have a low settling time-constant relative to the current reversal rate.
- **QHR, triple-series connection.** The relative change in the quantized Hall resistance using a triple-series connection in DC can be calculated from a mathematical model of the QHR [4]. In that reference an offset of less than 0.001 μΩ/Ω was found with typical leads and longitudinal resistance.

Simulation

We performed some simulations of the effect of each component in the measurement and the combined standard uncertainty for different combinations of resistors, as shown in Fig. 2.

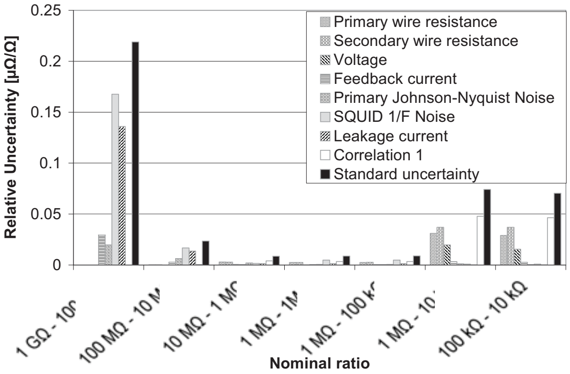

Figure 2. Standard uncertainty calculation and relative uncertainty produced by the most important components in normal situation of measurements. Correlation 1 represents the correlation between the primary and the standard wire resistance.

Conclusion

We estimate a combined standard uncertainty of order 0.25 μΩ/Ω for resistors of 10 kΩ to 1 GΩ with 10 V bridge voltage. The direct measurement of a 10 MΩ or 1 MΩ resistor with the QHR yields a combined standard uncertainty of 0.03 μΩ/Ω with 1 V. This shows that the two-terminal CCC is a powerful tool for high resistance scaling.

References

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